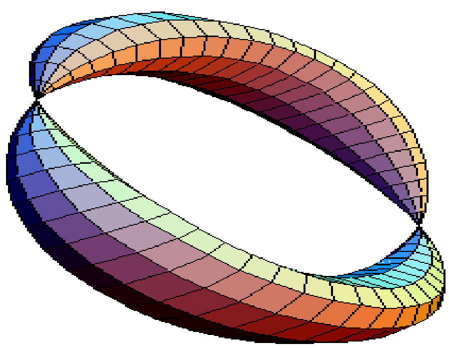
APPLIED MATHEMATICS HONOURS

Student Handbook 2025



School of Mathematics and Statistics

The University of Sydney

Contents

1 General Information

You should start with a general information about honours studies at the University of Sydney, the Faculty of Science web page honours in Science, and the general overview of honours in the School of Mathematics and Statistics.

1. **Honours in Applied Mathematics**

   Honours in Applied Mathematics is a one-year program consisting of four 6 credit point units of study and 24 credit points of research project. For more information about the structure and completion requirements of Applied Mathematics honours program see Table A.

2. **Project supervision**

   The candidate is required to find a prospective supervisor from among the the School of Mathematics and Statistics staff, who is agreeable to supervise the candidate's project in the candidate's chosen topic. Students are required to submit to the Honours Coordinator the Expression of Interest form approved by their supervisor before submitting the honours application to the Faculty of Science. The form signed by the honours coordinator should be attached to their online honours application.

3. **Entry requirements**

   Current students at the University of Sydney may apply for admission to Bachelor of Advanced Studies (Honours) (Mathematics (Applied)) or Bachelor of Science (Honours) (Mathematics (Applied)) depending on whether they completed two majors or one major. The Faculty requirements which must be met include:

   - qualifying for a degree in a major which is cognate to the proposed honours stream, that is, a major (or two majors) which provides a suitable background for the honours stream. In borderline cases the decision of whether a major is cognate is in the hands of the relevant Honours Coordinator and the Faculty of Science;
   - having a WAM of at least 65;
   - **securing the agreement of a supervisor.**

   In addition, the School of Mathematics and Statistics may require that the student has a total of at least 18 or 24 credit points (depending on their major requirement) of relevant 3000-level units of study in which:

   - the average mark of advanced level units of study is at least 65;
   - the average mark of mainstream level units of study is at least 75.

   If you have completed a mix of advanced and mainstream units where some are above and some below the thresholds, if you are not sure which of your courses are relevant, or if your average is just on the wrong side of the threshold, you can seek further advice from the relevant Honours coordinator.

4. **Students from other institutions**

   Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above and may apply for admission into Bachelor of Science (Honours) (Mathematics (Applied)).

5. **Online honours applications to the Faculty of Science**

   Application and enrolment information should be obtained from the Faculty of Science from their websites for current BSc or BSc/BAS students at the University of Sydney and for students from other institutions.

   – The Faculty of Science standard application deadline for honours commencing in Semester 1 is 15 January and for honours commencing in Semester 2 it is 25 June.
   – All acceptances into honours (including in cases where the schools requirements are not met) are ultimately at the discretion of the School. However, a student meeting all of the above criteria should be confident of acceptance.

6. **Honours scholarships**

   For scholarships available to honours students, see the website.

For further details, you may contact the honours coordinator

Prof Marek Rutkowski: Carslaw 814; marek.rutkowski@sydney.edu.au

2 Important Dates in 2025

Semester 1: 24 February to 1 June

- **Seminar presentation: Thursday, 17 April (week 8)**
- **Project submission: Monday, 26 May (week 13)**
  For students completing in Semester 1, 2025. An electronic file (pdf format) must be uploaded on Canvas before the deadline.
- **Examination period: 10–20 June**

Semester 2: 4 August to 9 November

- **Seminar presentation: Friday, 26 September (week 8)**
- **Project submission: Monday, 3 November (week 13)**
  For students completing in Semester 2, 2025. An electronic file (pdf format) must be uploaded on Canvas before the deadline.
- **Examination period: 17–29 November**

3 Applied Mathematics Honours

Independent research can be a life changing experience. In this honours program you will complete a research project in the discipline of Applied Mathematics. Together with your supervisor, you will identify a novel research question and develop a model, or propose some mathematical or computational analysis. You will then carry out this program of work to produce results that can be interpreted in terms of the underlying real-world problem. Your work will be assessed by a twenty minute presentation towards the end of your honours year and a 40 to 60 page honours thesis. Successful completion of your honours will clearly demonstrate that you have mastered significant research and professional skills for either undertaking a PhD or any variety of future careers.

3.1 Honours structure

Honours in Applied Mathematics consists of four 6-credit point coursework units including two core units in Applied Computational Mathematics and Advanced Methods in Applied Mathematics. Students will also complete 24-credit points of research project.

3.2 Coursework (24 credit points)

1. **Core units (12 credit points)**
   Students are required to complete the following core units of study:
   - Semester 1: MATH 4411 Applied Computational Mathematics
   - Semester 2: MATH 4412 Advanced Methods in Applied Mathematics
2. **Selective units (12 credit points)**
   In addition to two core units, students should choose their two selective units from units offered by the School of Mathematics and Statistics listed as 4000 level or higher, which have not already been taken for credit with the proviso that at most one unit labelled 5000 or higher may be taken. For the full list of selective units available to honours students in Applied Mathematics, see Table A. Your selection of units should be discussed with your supervisor and you should get their approval of your choice of selective units.

3.3 Honours project AMAT4103–4106 (24 credit points)

Each student is required to complete an honours project (composed of a written thesis and a seminar presentation) on an Applied Mathematics topic, under the supervision of a member of staff of the School of Mathematics and Statistics. Students should enrol in two project units in each semester: AMAT4103–4104 in their semester 1 and AMAT4105–4106 in their final semester.

3.4 Applied Mathematics honours coordinator in 2025

Prof Marek Rutkowski
Room 814, Carslaw Building
Email: marek.rutkowski@sydney.edu.au

4 Core Units of Study

MATH4411 Applied Computational Mathematics *(Semester 1)*

Computational mathematics fulfils two distinct purposes within Mathematics. On the one hand the computer is a mathematicians laboratory in which to model problems too hard for analytical treatment and to test existing theories; on the other hand, computational needs both require and inspire the development of new mathematics. Computational methods are an essential part of the tool box of any mathematician. This unit will introduce you to a suite of computational methods and highlight the fruitful interplay between analytical understanding and computational practice.

In particular, you will learn both the theory and use of numerical methods to simulate partial differential equations, how numerical schemes determine the stability of your method and how to assure stability when simulating Hamiltonian systems, how to simulate stochastic differential equations, as well as modern approaches to distilling relevant information from data using machine learning. By doing this unit you will develop a broad knowledge of advanced methods and techniques in computational applied mathematics and know how to use these in practice. This will provide a strong foundation for research or further study.

MATH4412 Advanced Methods in Applied Mathematics *(Semester 2)*

Much of our physical world is nonlinear. If you take two rulers and place one on top of another, the height of the combined object is the sum of the individual heights of each ruler. But whether you are looking at herds of bisons in a landscape, the viral load in an infective patients bloodstream, or the interaction of black holes far away in the universe, it turns out the sum of individual components does not necessarily give a true measure of reality. To describe these systems, we need methods that apply to nonlinear mathematical models.

This course will cover theoretical methods (some exact, some in limits and others that are qualitative) to describe, solve and predict the results of such models. Classical mathematical methods were developed for linear models. We will start with building blocks to describe models of semi-classical quantum mechanics and related orthogonal polynomials. These turn out to be generalizable to models that arise in modern physics, such as quantum gravity and random matrix theory. These lead naturally to integrable systems.

5 Choice of Selective Units of Study

In addition to the core units, honours students in Applied Mathematics should choose their two selective units from units offered by the School of Mathematics and Statistics listed as 4000 level or higher, which have not already been taken for credit with the proviso that at most one unit labelled 5000 or higher may be taken. Your choice of selective units of study should always be done in consultation with a supervisor of your research project.

For the full list of selective units available to students enrolled in Applied Mathematics honours and specific rules governing your honours studies, see Table A.

6 SCIE4999 Final Honours Mark Unit

SCIE4999 Final Honours Mark Unit. All students in Science Honours must enrol in this non-assessable unit of study in their **final semester** (but not before the final semester). This unit will contain their final Honours mark as calculated from the coursework and research project units (50% each).

Applied Mathematics honours students should enrol in AMAT4103 and AMAT 4104 in the first semester of their honours year and in AMAT4105 and AMAT4106 in the second semester. They should be correctly enrolled in their final project unit AMAT4106 and SCIE4999 in their projected final semester.

N.B.: Changes to enrolments into AMAT4106 and SCIE4999 must be made before the census date and will only be approved if they are the "fault" of the university. A student's error is not an acceptable reason. Wording of forms must represent how this was the error of the school/faculty/university or it will not be accepted by the fees team.

7 Marking Scale for Honours

The final honours mark SCIE4999 for each student is based on the following marking scheme:

- **50% for the project unit AMAT4103–4106,**
- **50% for four units of study (12.5% for each).**

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The project will be marked with this scale in mind.

GRADE OF HONOURS	FACULTY-SCALE
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	50–69
Fail	00–49

8 Honours Research Project in Applied Mathematics

A significant part of the honours year is the completion of a research project by each student. Each student must choose a project supervisor who is willing to supervise the student's chosen topic for the project. Project topics and supervisors should be finalised by the beginning of the first semester, so that students can commence work immediately on their projects. The following list shows the main Applied Mathematics research areas:

- Dynamical systems
- Geophysical and astrophysical fluid dynamics
- Industrial and biomedical modelling
- Integrable systems
- Mathematical biology

Members of the Applied Mathematics Research Group in 2025: Anna Aksamit, Eduardo Altmann, Harini Desiraju, Nathan Duignan, Holger Dullin, Ben Goldys, Georg Gottwald, Nalini Joshi, Peter Kim, Christopher Lustri, Robert Marangell, Mary Myerscough, Milena Radnović, Pieter Roffelsen, Marek Rutkowski, Ayesha Sohail, Sharon Stephen, Martin Wechselberger, Caroline Wormell, Ding-Xuan Zhou, Zhou Zhou.

Their email addresses and research interests can be found here.

8.1 Project assessments

The written thesis will be marked by three examiners and each marking will therefore constitute 30% of the final mark from the project unit.

The final mark from the project unit AMAT4103–4106 will be awarded according to the following marking scheme:

- **90% for a written thesis,**
- **10% for a seminar presentation on the project.**

The seminar is an opportunity for each student to present the material of her/his research project to members of Applied Mathematics Research Group. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The presenter of the best talk will be awarded the Chris Cannon Prize. Marks for the thesis and seminar presentation will be awarded for:

(i) selection and synthesis of source material;
(ii) evidence of understanding;
(iii) evidence of critical ability;
(iv) clarity, style and presentation;
(v) mathematical and/or modelling expertise.

8.2 Project guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report is to be submitted to the Honours Coordinator. This report includes a half page description about the students aim/scope of the project and a half page description about what the student has achieved in Semester 1 and what the student wants to achieve in Semester 2. This report has to be approved by the supervisor before submission.
- The project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- The thesis should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand. The work must contain substantial mathematical content.
- Students are recommended to use LaTeX in typesetting their projects. Additional information on LaTeX can be found here.
- The length of the written thesis should be between 40 to 60 typed (normal LaTeX font size) A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the project exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.
- Computer programs essential to the work should be included (with adequate commentary) as additional material in appendices.
- Students should be careful to provide full and correct referencing to all material used in the preparation of projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless reinforcing an important point.
- Three examiners will be appointed to assess each written thesis. Although marking schemes may differ, the assessment of the thesis will be based on:

  (i) selection and synthesis of source material;
  (ii) evidence of understanding;
  (iii) evidence of critical ability;
  (iv) clarity, style and presentation;
  (v) mathematical and/or modelling expertise.

- Students who have worked on their project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their thesis.

8.3 Recent honours theses in Applied Mathematics

- Hamish Blair: Microlocal analysis and the geometry of distributions.
- Nicholas Cranch: Customising the stability of truncation schemes.
- Kai Nielson: The effects of partial slip on the stability of the rotating disc boundary layer.
- Jessica Slegers: Harnack-type inequalities for nonlinear evolution equations
- Samuel Lin: Symmetries and exact solutions of PDEs.
- Chamal Perera: Orbits of a small satellite co-orbiting around a Lagrange point 1 space station.
- Timothy Lapuz: A geometric approach to transonic accretion flows: Stars and black holes.
- Andre Nunez: Optimal sex allocation for dioecious species.
- Viney Kumar: How does the evolution of monogamy depend on human life history? A mathematical model.
- Kenny Chen: Partial difference equations on face centred cubic lattices.
- Charles Lilley: Optimising a novel combination cancer treatment.
- Emily Cooper: Modelling refugee behaviour.
- Mia Bridle: Spectral analysis of coupled and decoupled eigenvalue problems.
- Elizabeth Rose: The vortex filament equation.
- Yige Bian: Mathematical modelling of cancer immunotherapy.
- Ren Li: Optimal tensor network decoder for fault-tolerant quantum error correction of the XZZX surface code.
- Patrick Cahill: Who's going to win? Modelling elections with an adapted Hegselmann-Krause model and data assimilation.
- Harry Hiatt: A traveller's guide to billiard knots.
- Yuan Xu: A lipid-structured model of atherosclerosis with smooth muscle cell-derived macrophages.
- Ivan Hu: Glycolytic oscillations in the integrated oscillator model.
- Charlotte Banbury: Mathematical modelling of T cell and cancer cell dynamics in the immune system.
- Kate Fiumara: Modelling of T cell affinity maturation.

- Ewan Macfarlane: Null geodesic trajectories of photons around Schwarzschild black holes with Hamiltonian methods: The Halo Drive.
- Lei Qu: Balancing exploration and exploitation: Exploratory optimal control and stopping time.
- Georgia Wang: Mathematical modelling of tumour-immune dynamics in combination checkpoint blockade therapy.

8.4 Proposed project topics in 2025

You will find below a list of proposed project topics for honours students in Applied Mathematics in 2025. Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible. The list is not exhaustive and thus you may wish to suggest your own topic for the project or discuss any other topic with a potential supervisor. Notice that each student must find a member of staff who will agree to supervise the project before applying for admission to Applied Mathematics honours.

Meso-scale structures in complex networks

Prof E. Altmann; Carslaw 526; eduardo.altmann@sydney.edu.au; phone 9351-4533

Characterizing the main statistical features of complex networks is an important step in the study of a variety of systems, from social media to ecology. One of the most successful approaches is to identify groups of nodes with similar connectivity patterns to obtain an intermediate, meso-scale, description of the network as a whole. The goal of this project is to explore data-analysis methods for the detection of such structures with mathematical models that reveal the mechanisms behind their formation.

Random walks in Monte Carlo

Prof E. Altmann; Carslaw 526; eduardo.altmann@sydney.edu.au; phone 9351-4533

Markov Chain Monte Carlo (MCMC) methods allow for efficient computational explorations of rare configurations that are widely used in Applied Mathematics, Data Science, and Machine Learning. The goal of this project is to study different MCMC methods and their suitability to address problems of interest in the study of problems of interest, which could focus on the sampling of infection trees in epidemics or on triangulation of manifolds in computational topology.

Fractals in transient chaos

Prof E. Altmann; Carslaw 526; eduardo.altmann@sydney.edu.au; phone 9351-4533

One of the most beautiful aspects of non-linear dynamics is that their long-time behaviour is often described by fractal sets. This happens not only in the famous case of chaotic attractors but also in dynamical systems in which the asymptotic dynamics is trivial, but the transient dynamics is chaotic. In this project we will investigate how such fractal sets appear in simple dynamical systems and how they change with the presence of perturbations, such as noise or dissipation in an otherwise Hamiltonian system.

Quantizing Painlevé equations.

Dr H. Desiraju; Carslaw 630; harini.desiraju@sydney.edu.au

Description: Painlevé equations are a class of integrable second order ODEs with an extraordinarily rich mathematical foundation, from their Hamiltonians to the associated geometry. As such, the problem of quantizing these equations is a multi-faceted one, in the sense that one could quantize one or more of their associated structures. These quantizations have recently appeared in several areas of mathematics and physics from random matrices to black hole physics. In this project we would study the quantization of Painlevé equations in one or more ways, using their Hamiltonians and associated linear problems for example.

Finding regularity in chaos

Dr N. Duignan; Carslaw 606; nathan.duignan@sydney.edu.au

Chaotic systems are identified by the unpredictability of their motions. Some examples include models of the weather, planetary motion, or the ion trajectories in a nuclear fusion reactor. Remarkably, these chaotic systems often have a subset of initial conditions which provide predictable, regular motion. Finding the initial conditions that lead to regular motion is essential to understanding the motion of a chaotic system. In particular, for nuclear fusion reactors, it is crucial to try and maximise the initial conditions that give regular motion to achieve a stable reactor. In this project, you will study techniques for detecting regions of chaos and apply it to an important system, for example, nuclear fusion reactors.

Numerical detection of integrability

Dr N. Duignan; Carslaw 606; nathan.duignan@sydney.edu.au

Integrable systems are identified by their complete predictability. Some examples include the two-body problem, some problems of rigid body dynamics, and magnetic fields constructed for optimal confinement of a plasma. The study of integrable systems lies on the intersection of dynamical systems, differential geometry, topology, algebra, and much more. Given a system with parameters, it is often important to know when this system is integrable. For example, in the case of confinement of a plasma, the magnetic field lines need to form an integrable system to ensure the plasma is confined. In this project, you will develop a technique for numerically finding when a value of the parameters gives an integrable system and apply it to important systems.

Optimal magnetic axes

Dr N. Duignan; Carslaw 606; nathan.duignan@sydney.edu.au A stellarator is a proposed device for the magnetic confinement of the plasma created in a nuclear fusion reaction. In a stellarator, the magnetic field lines form the shape of a twisted donut. At the centre of the stellarator lies a magnetic field line which closes on itself to form a loop, called the magnetic axis. In this project you will try to understand how the entire stellarator must look for good confinement of a plasma based purely off the shape of the magnetic axis. The project could involve methods from differential geometry, dynamical systems, Hamiltonian mechanics, Fourier analysis, and the theory of 3d curves.

Other possible research projects

Dr N. Duignan; Carslaw 606; nathan.duignan@sydney.edu.au

Other possible research projects under the supervision of Dr Duignan include topics in integrable systems, chaotic and regular dynamics, Hamiltonian mechanics, (pre)symplectic geometry, the n-body problem, plasma physics and toroidal confinement devices, normal form theory, and applications of each topic.

You can read more on https://www.maths.usyd.edu.au/u/nathand/. Please contact him if interested!

Chaotic dynamics of the pentagon

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

A chain of planar rigid bodies is a simple mechanical system with n segments connected by joints that allow free rotation. Connecting the first segment to the last by another joint gives a closed chain. Since the distance between the joints is fixed the closed chain has n degrees of freedom. Reduction by translations and rotations leaves n 3 degrees of freedom specifying the shape. For certain parameters the dynamics of this system is chaotic in the sense of Anosov. The goal of the project is to study the dynamics in the first non-trivial case of the pentagon (n=5). In a chaotic system periodic orbits are dense in phase space, and the goal of the project is to find and describe the periodic orbits of this system, using a combination of analytical and numerical tools.

Non-normality in the Hopf bifurcation

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

A real square matrix is normal if it commutes with its transpose. For example, orthogonal, symmetric, and skew-symmetric matrices are normal matrices. Non-normal matrices are important in regards to understanding stability and instability in dynamical systems. For example, certain types of instabilities in fluid dynamics can be explained using non-normal matrices. The Hamiltonian Hopf bifurcation describes the bifurcations that can occur when two pairs of imaginary eigenvalues collide on the imaginary axis and branch off into the complex plane. Non-normal matrices appear naturally near this bifurcation. The goal of the project is to describe and analyse the transient growth associated to non-normality as it appears near this bifurcation.

Symplectic integration of the regularised planar circular restricted 3 body problem

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

The restricted three body problem describes the motion of a test particle in the field of two heavy masses rotating around each other in circular orbits. The problem has a singularity when the test particle collides with either of the other masses. The collision can be regularised, such that the solutions are defined for all times. The goal of the project is to construct and implement a symplectic integration method for the regularised problem. This can then be used to study periodic orbits in this chaotic system, in particular collision orbits. The study of collision orbits is interesting and relevant because double collision orbits describe the motion of a spacecraft from one body to the other.

Stochastic games with incomplete information

Prof B. Goldys; Carslaw 709; beniamin.goldys@sydney.edu.au; phone 9351-2976

Stochastic games arise in various problems of mathematical finance, economics, social sciences and biology. They also provide tools to study certain deterministic partial differential equations, such as p-Laplace equation and curvature flows. In many problems arising in sciences players do not have complete information about the game but still must make decisions. The aim of this project is to study this little understood problem.

Magnetic nanowires

Prof B. Goldys; Carslaw 709; beniamin.goldys@sydney.edu.au; phone 9351-2976

Ferromagnetic materials are naturally divided into micro-meter size domains, each of which has a magnetic moment (spin) that can be represented by a unit vector pointing "up" or "down." This allows for coding information and for this reason ferromagnetic materials provide a widely used basis for the design of magnetic memories. Behaviour of a large system of spins is modelled using a system of partial differential equations closely related to certain equations arising in differential geometry (harmonic maps). It was observed by physicists that imperfections of the material and the so-called thermal noise can destroy the coded information, so that understanding impact of noise is crucial for design of magnetic memories. A one-dimensional nanowire is frequently used as a model of real wire of positive width. To what extent this approximation is valid in the presence of noise is an open problem we will investigate.

Stochastic differential equations on groups of matrices

Prof B. Goldys; Carslaw 709; beniamin.goldys@sydney.edu.au; phone 9351-2976

We will start with learning about SDEs on groups of matrices in general, but we will mainly focus on the Lorentz group $PSO(1;d)$, which is the linear isometry group of Minkowski spacetime M. In particular, this will allow us to give a meaning to the concept of Markov process on M. The main problem will be to describe the long time behaviour of solutions.

Data-driven modelling: Finding models for observations in finance and climate

Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

When given data, which may come from observations of some natural process or data collected form the stock market, it is a formidable challenge to find a model describing those data. If the data were generated by some complex dynamical system one may try and model them as some diffusion process. The challenge is that even if we know that the data can be diffusive, it is by no means clear on what manifold the diffusion takes place. This project aims at applying novel state-of-the-art methods such as diffusion maps and nonlinear Laplacian spectral analysis to determine probabilistic models. You will be using data from ice cores encoding the global climate of the past 800kyrs as well as financial data. In the latter case you might be able o recover the famous Black-Scholes formula (but probably not). This project requires new creative ideas and good programming skills.

Optimal power grid networks and synchronisation

Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Complex networks of coupled oscillators are used to model systems from pacemaker cells to power grids. Given their sheer size we need methods to reduce the complexity while retaining the essential dynamical information. Recent new mathematical methodology was developed to describe the collective behaviour of large networks of oscillators with only a few parameters which we call "collective coordinates." This allows for the quantitative description of finite-size networks as well as chaotic dynamics, which are both out of reach for the usually employed model reduction methods.

You will apply this methodology to understand causes of and ways to prevent glitches and failure in the emerging modern decentralised power grids. As modern societies increase the share of renewable energies in power generation the resulting power grid becomes increasingly decentralised. Rather than providing a power supply constant in time, the modern decentralised grid generates fluctuating and intermittent supply. It is of paramount importance for a reliable supply of electric power to understand the dynamic stability of these power grids and how instabilities might emerge. A reliable power-grid consists of well-synchronised power generators. Failing to assure the synchronised state results in large power outages as, for example, in North America in 2003, Europe in 2006, Brazil in 2009 and India in 2012 where initially localised outages cascade through the grid on a nation-wide scale. Such cascading effects are tightly linked to the network topology. Modern power-grids face an intrinsic challenge: on the one hand decentralisation was shown to favour synchronisation in power grids, on the other hand decentralised grids are more susceptible to dynamic perturbations such as intermittent power supply or overload.

The project uses analytical methods as well as computational simulation of models for power grids. You will start with a simple network topology and then, if progress is made, use actual power grid topologies.

Data assimilation in numerical weather forecasting and climate science

Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Data assimilation is the procedure in numerical weather forecasting whereby the information of noisy observations and of an imperfect model forecast with chaotic dynamics, we cannot trust, is combined to find the optimal estimate of the current state of the atmosphere (and ocean). Data assimilation is arguably the most computationally costly step in producing modern weather forecasts and has been topic of intense research in the last decade. There exists several approaches, each of which with their own advantage and disadvantages. Recently a method was introduced to adaptively pick the best method to perform data assimilation. This method employs a switch which, although it seems to work, has not been linked to any theoretical nor physical properties of the actual flow. This project will be using toy models for the atmosphere to understand the witch with the aim of improving the choice of the switching parameter.

Networks of coupled oscillators

Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Many biological systems are structured as a network. Examples range from microscopic systems such as genes and cells, to macroscopic systems such as fireflies or even an applauding audience at a concert. Of paramount importance is the topography of such a network, i.e., how the nodes, let's say the fireflies, are connected and how they couple. Can they only see their nearest neighbours, or all of them. Are some fireflies brighter than others, and how would that affect the overall behaviour of a whole swarm of fireflies? For example, the famous 'only 6 degrees of separation'-law for the connectivity of human relationships is important in this context.

In this project we aim to understand the influence of the topography of such a network. Question such as: How should a network be constructed to allow for maximal synchronization will be addressed. This project requires new creative ideas and good programming skills.

Other possible projects

Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

I work on numerous topics. Please check out my webpage. The publication list will show you topics I am interested in. If any of those topics grab your interest or if you have your own idea for a topic, please talk to me.

Discrete soliton equations

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Famous PDEs such as the Korteweg-de Vries equation (which have soliton solutions) have discrete versions (which also have soliton solutions). These discrete versions are equations fitted together in a self-consistent way on a square, a 3-cube or an N-dimensional cube. These have simple, beautiful geometric structures that provide information about many properties: solutions, reductions to discrete versions of famous ODEs, and deeper aspects such as Lagrangians. This project would consider generalisations of such structures and/or properties of the solutions, such as finding their zeroes or poles.

Integrable discrete or difference equations

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

The field of integrable difference equations is only about 20 years old, but has already caused great interest amongst physicists (in the theory of random matrices, string theory, or quantum gravity) and mathematicians (in the theory of orthogonal polynomials and soliton theory). For each integrable differential equation there are, in principle, an infinite number of discrete versions. An essay in this area would provide a critical survey of the many known difference versions of the classical Painlevé equations, comparisons between them, and analyse differing evidence for their integrability. Project topics would include the derivation of new evidence for integrability. The field is so new that many achievable calculations remain to be done: including derivations of exact solutions and transformations for the discrete Painlevé equations.

Exponential asymptotics

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Near an irregular singular point of a differential equation, the solutions usually have divergent series expansions. Although these can be 'summed' in some way to make sense as approximations to the solutions, they do not provide a unique way of identifying a solution. There is a hidden free parameter which has an effect like the butterfly in chaos theory. This problem has been well studied for many classes of nonlinear ODEs but almost nothing is known for PDEs and not much more is known for difference equations. This project would include studies of a model PDE, like the famous Korteweg-de Vries equation near infinity, or a difference equation like the string equation that arises in 2D quantum gravity.

Cellular automata

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Cellular automata are mathematical models based on very simple rules, which have an ability to reproduce very complicated phenomena. (If you have played the Game of Life on a computer, then you have already seen automata with complicated behaviours.) This project is concerned with the mathematical analysis of their solutions, which lags far behind corresponding developments for differential or difference equations.

In this project, we will consider a family of cellular automata called parity filter rules, for which initial data are given on an infinite set. For example, consider an infinitely long train of boxes, a finite number of which have a ball inside, whilst the remainder are empty. At each time step, there is a simple rule for moving the leftmost ball in a box to the next empty box on the right. Continue until you have finished updating all nonempty boxes in the initial train. (Try this out for yourself with adjacent boxes with three balls, followed by two empty boxes and then two boxes with balls inside. What do you see after one update? Two updates?) It turns out that these box-and-ball systems replicate solitons, observed in solutions of integrable nonlinear PDEs. In this project, we will consider how to derive parity filter rules from nonlinear difference equations, and how to analyse their solutions. One direction for the project is to analyse the solutions as functions of initial data. Another direction is to develop ways to describe long-term behaviours.

Modelling the evolution of human post-menopausal longevity and pair bonding

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

A striking contrast between humans and primates is that human lifespans extend well beyond the end of the female reproductive years. Natural selection favours individuals with the greatest number of offspring, so the presence of a long female post-fertile period presents a challenge for understanding human evolution.

One prevailing theory that attempts to explain this paradox proposes that increased longevity resulted from the advent of grandmother care of grandchildren. We have developed preliminary age-structured PDE models and agent-based models to consider the intergenerational care of young proposed by this Grandmother Hypothesis. The project will involve extending the models to consider whether the presence of grandmothering could increase the optimum human longevity while simultaneously maintaining a relatively early end of fertility as seen in humans (and killer whales).

Analytical approaches will involve developing numerical schemes for the PDEs and analytically and numerically studying the steady state age distributions and growth rates of the populations with and without grandmothering and under different life history parameters, e.g. longevity and end of fertility.

We have now also begun to explore mating strategies, especially pair bonding, yet another unique human characteristic among mammals. Speculations about how pair bonding developed from our ancestral roots abound and are open to being quantified, modelled, and analysed. Like the grandmothering models, these investigations will involve PDEs or agent-based models.

Modelling cancer immunotherapy

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

A next generation approach to treating cancer focuses on cancer immunology, specifically directing a person's immune system to fight tumours. Recent directions in cancer immunotherapy include

- Oncolytic virotherapy: infecting tumours with genetically-engineered viruses that preferentially destroy tumour cells and induce a local anti-tumour immune response,
- Preventative or therapeutic cancer vaccines: stimulating a person's immune system to attack tumour colonies to prevent or hinder tumour development,
- Cytokine therapy: using immunostimulatory cytokines to recruit immune cells and enhance existing anti-tumour immune responses.

These treatments can be used alone or in combination with each other or with other forms of treatment such as chemotherapy. Since immunotherapy often involves immune responses against small tumours, often close to inception, they are highly spatially dependent and often probabilistic. The goal of the will be to develop differential equation and possibly probabilistic agent-based models to understand the tumour-virus-immune dynamics around a small, developing tumour and determine conditions that could lead to effective tumour reduction or complete elimination. The project will involve developing the models and schemes for numerically simulating the ODE and PDE systems, and if possible, performing a stability analysis of the ODE system.

Modelling liver disease and risk factors

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

(In collaboration with Dr Joachim Worthington, The Daffodil Centre)

Fatty liver disease can decrease quality-of-life and lead to significant risk of developing liver cancer. Fatty liver disease is usually caused by weight gain, excess alcohol intake, or other metabolic factors such as diabetes. For people with fatty liver disease, routine monitoring to detect development of late-stage liver disease or cancer as early as possible can lead to better health outcomes. However, for many this is not necessary as their risk is relatively low. In particular, alcohol cessation and weight loss can quickly reduce risk of liver disease and cancer, meaning ongoing monitoring has limited benefits.

The goal of this project will be to analyse existing models of liver disease, and analyse and potentially adapt them to capture the impact of risk factor changes over time, such as weight gain/loss and alcohol use/cessation. This will include understanding the dynamics of these models and introducing time-dependent parameters. You could also look at potential data sources that could inform the impact of changes to risk factors, such as data from mice model studies. This project would suit someone with coding/numerical analysis experience and a potential interest in epidemiology/public health.

References:

- Holzhütter et al., *Mathematical modeling of free fatty acid-induced non-alcoholic fatty liver disease (NAFLD)*, biorxiv, 2020.
- Attia, *Novel approach of multistate Markov chains to evaluate progression in the expanded model of non-alcoholic fatty liver disease*, Frontiers in Applied Mathematics and Statistics, 2022.

Estimating the potential impact of lead-time bias in epidemiological PDE modelling

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

(In collaboration with Dr Joachim Worthington, The Daffodil Centre)

In epidemiology and public health, predictive models are often used to “chain together” multiple events that can occur across a person’s lifetime. Typically, the data to inform the time between these events comes from trial data. However, trial data on a disease of interest does not start from the time a person develops a disease, but from the time they were recruited to the trial, which may be long after. This is related to *lead-time bias*, which is a statistical bias that occurs when observing a disease detected by screening, which will naturally be diagnosed earlier than a disease detected by the onset of symptoms. Understanding these biases can improve epidemiological modelling.

This project will review existing approaches and sources to estimating lead-time biases, and use simple existing models of liver disease to estimate the eventual of these biases on health outcomes. This could involve exploring Bayesian models of lead time bias, or expanding on the use of Kolmogorov backward equations in development of epidemiological models. This project would suit someone with coding/numerical analysis experience and a potential interest in epidemiology/public health.

References:

- Wu et al., *Bayesian inference for the lead time in periodic cancer screening*, Biometrics, 2020.
- Ge et al., *Estimating lead-time bias in lung cancer diagnosis of patients with previous cancers*, Statistics in Medicine, 2021.
- Worthington et al, *An economic evaluation of routine hepatocellular carcinoma surveillance for high-risk patients using a novel approach to modelling competing risks*, medrxiv, 2024.

Numerical rational approximation and asymptotic analysis

Dr C. Lustri; Carslaw 627; christopher.lustri@sydney.edu.au; phone 9351-3879

Asymptotic methods are mathematical techniques that allow us to analytically approximate the solutions to highly complex problems. I am particularly interested in using these methods to study nonlinear waves and fluid dynamics. A significant limitation on asymptotic methods is that they typically require some nice leading-order approximation of system behaviour, which is not always possible. In this project, we will explore the behaviour of complex model systems by replacing nice leading-order approximations with numerically approximated rational function approximations, such as Pade or AAA approximations, and develop hybrid asymptotic methods for complex systems.

Nonlinear waves and bifurcations in difference equations and discrete lattices

Dr C. Lustri; Carslaw 627; christopher.lustri@sydney.edu.au; phone 9351-3879

Dr C. Lustri; Carslaw 627; christopher.lustri@sydney.edu.au; One of the most significant challenges in the study of discrete systems is combining continuous long-wave approximations with essentially discrete phenomena. If we naively apply continuum approximations, we can lose discrete-scale behaviour, such as discrete bifurcations. Projects in this category involve applying discrete multiple scales techniques to study systems such as discrete nonlinear Schrodinger equations in order to predict the shapes of the impulse, the onset of catastrophic behaviour, and the effect of integrable and non-integrable discretizations.

Stokes phenomenon in PDEs

Dr C. Lustri; Carslaw 627; christopher.lustri@sydney.edu.au; phone 9351-3879

Stokes phenomenon describes rapidly-changing asymptotic behaviour in the solutions to differential equations, which is typically hidden from conventional asymptotic analyses. Methods known as exponential asymptotics have been used to capture these effects, and Stokes phenomenon in ordinary differential equations is a relatively well-understood problem. Stokes phenomenon in partial differential equations is much more complicated and has only been studied in relatively simple cases. These methods require us to understand the behaviour of the PDE solution in the complex plane. This project involves learning about exponential asymptotic methods, and applying them to approximate the solutions to model PDEs from wave dynamics.

Modelling self-assembly in ant populations

Dr C. Lustri; Carslaw 627; christopher.lustri@sydney.edu.au; phone 9351-3879

Ants in the wild can assemble into complicated structures including bridges, chains, nests, and boats. This project aims to develop models for ant behaviour and interactions that can be used to predict properties of structures created by spontaneous self-assembly. This project will explore the construction and predict the mechanical properties of hanging chains of ants as they lift leaves with different rigidity, and attempt to determine whether ants optimise their behaviour to build efficient structures based on local information alone.

Geometric aspects of Turing bifurcations

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

The Turing bifurcation is a classical example of a diffusion driven instability. This project will attempt to look at some geometric aspects of the system at the onset of a Turing bifurcation, as well as potential factorisation of the system when the diffusion is either extremely large or extremely small.

Fast-slow splitting in characteristic determinants

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Eigenvalue problems on a finite interval can often be characterised in terms of the vanishing of the determinant of a matrix. Such a determinant is called the characteristic determinant of the system. When multiple time-scales are present, this often results in the ability to factor the characteristic determinant into characteristic determinants of lower-dimensional systems. This project will look at how this factorisation takes place, based on the entries of the original system.

Symmetries in PDEs

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

This project is about the relationship between symmetries of partial differential equation, the coordinate systems in which the equation admits solutions via separation of variables and the properties of the special functions that arise in this manner. A major focus of this project lying at the intersection of geometry, algebra and analysis is the characterisation of separable coordinate systems in terms of the second-order symmetry operators of for the equations.

Chemotaxis in models with zero/negative diffusivity

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Chemotaxis is the movement of a cell via advection either towards or away from a chemical source. It has been used in many biological models, from slime-moulds to motile bacteria, to roadway construction by humans. Typically linear diffusivity has been studied, but lately models where the diffusivity is allowed to change sign have become of interest. This project will examine the existence of travelling wave solutions in such models, as well as some elementary stability properties of such solutions.

Stability in a model of herd grazing and chemotaxis

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

This project will examine a model of the formation of a herd of grazing animals. The model will focus on two major factors, how the animal seeks food and how the the animals interact with each other. Remarkably, the model shares many properties with another, well studied model, that of so-called bacterial chemotaxis. The aim of this project will be to analyse, both numerically and analytically, such a model, and to understand certain special solutions in the model, called travelling waves, as well as their stability.

Absolute spectrum of St. Venant roll waves

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Roll waves are a phenomenon that occurs when shallow water flows down an inclined ramp. Mathematically they can be modelled by the St. Venant equations. Typically roll waves occur as periodic solutions, however if they are far enough apart, they can be treated as solitary waves. In this case, the spectrum of the linearised operator governs their dynamics, and in particular, their stability properties. This project will focus on computing the absolute and essential spectrum of these solitary waves. Medium computational skills are required for this project.

Other possible research topics

A/Prof R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Other projects under the supervision of Dr Marangell include topics in the areas of nonlinear standing or travelling waves, topics in the application of geometric and topological methods in dynamical systems and PDEs, symmetries in ODEs and PDEs and other research topics in the history of mathematics and science in general. Examples of nonlinear standing and travelling waves come from models in a wide range of areas which include mathematical biology, chemistry and physics. More specific examples would be standing/travelling waves in population dynamics, combustion models, and quantum computing, but really there are many, many examples, so please contact Dr Marangell for further details.

PDE models for the distribution of ingested lipids in macrophages in atherosclerotic plaques

Prof M. R. Myerscough; Carslaw 626; mary.myerscough@sydney.edu.au; phone 9351-3724

Atherosclerotic plaques are accumulations of lipid (fat) loaded cells and necrotic (dead) cellular debris in artery walls. They are caused by LDL (which carries 'bad cholesterol') penetrating the blood vessel wall, becoming chemically modified (usually oxidised) and setting off an immune reaction. In response to this immune reaction, macrophages (a type of white blood cell) enter the artery wall and consume the modified LDL. In this way macrophages accumulate lipids and as more LDL and more cells enter the vessel walls the population of cells also grows. Other processes can affect the growth or regression of the plaque, such as cell death, cells leaving the tissue and lipid export from inside cells to HDL (which carries 'good cholesterol' which is good because it's been carried away from the plaque). When atherosclerotic plaques grow very large and rupture they can cause heart attacks and strokes which are one of the two leading causes of death in the developed world. (The other is cancer.)

We have written a partial differential equation model for the accumulation of cells and lipids in plaques. In this model, the number of macrophages in the plaque is a function of both time t and accumulated lipid a. The primary equation is an advection equation with nonlinear source and sink terms, including a term with an integral convolution that models what happens when macrophages phagocytose (=eat) other macrophages that are dead or dying.

We have done an analysis of this model at steady state when all the processes (lipid ingestion, macrophages leaving the plaque, the action of HDL) occur at a constant rate. This project will build on this analysis and has the aim of producing numerical solutions to the model when model processes are functions of a, the accumulated lipid inside the cell. This project is particularly suitable for students who are interested in applications of mathematics to biomedical problems, have completed a third year unit on PDEs and have at least some experience in coding in Matlab, C, Python or similar.

Mathematical billiards

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

Mathematical billiards have been an established topic for research for about one century. They have application in any situation involving collisions and reflections. They are used as a model for the popular game of billiards, and also in laser techniques, the statistical interpretation of the second law of thermodynamics wind-tree model, the dynamics of ideal gas, tri-atomic chemical reactions etc. The field of mathematical billiards is at the cutting edge of mathematics research, and work in the field is highly valued: several Fields Medals were recently awarded for contributions in the area. The research on this project can vary from making computer simulations to more theoretical work. Writing an essay is also available.

Poncelet porisms

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

Suppose that two conics are given in the plane, together with a closed polygonal line inscribed in one of them and circumscribed about the other one. The Poncelet porism states that then infinitely many such closed polygonal lines exist and all of them with the same number of sides. That statement is one of most beautiful and deepest contributions of the 19th century geometry and has many generalisations and interpretations in various branches of mathematics. In this essay, the student will present rich history and current developments of the Poncelet porism.

Elliptical billiards and their periodic trajectories

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

We consider billiards in a domain bounded by arcs of several conics belonging to a confocal family. When the boundary of such a billiard does not contain reflex angles, the system turns out to be integrable. Geometrically, the integrability has the following manifestation - for each billiard trajectory, there is a curve, called caustic, which is touching each segment of the trajectory. For elliptical billiards, the caustics are conics from the same confocal family. Integrability implies that the trajectories sharing the same caustic are either all periodic with the same period or all non-periodic.

On the other hand, if there is at least one reflex angle on the boundary, the integrability will be broken, although the caustics still exist. Such billiards are thus called pseudo-integrable and there may exist trajectories which are non-periodic and periodic with different periods sharing the same caustic.

An essay on this topic would provide a review of classical and modern results related to the elliptical billiards. In a project, the student would explore examples of billiard desks.

Optimizing solar energy systems: Integrating heat equation modelling and machine learning for enhanced efficiency and sustainability

Dr A. Sohail; Carslaw 708; ayesha.sohail@sydney.edu.au

SDG challenges of greener energy production can be addressed through mathematical modelling and machine learning techniques. The heat equation, for instance, helps model temperature distributions and their impact on solar panel efficiency and system performance. When combined with machine learning, these models can be further enhanced. Machine learning algorithms can analyse historical temperature and performance data to optimize thermal management strategies, predict potential overheating issues, and refine system designs. By integrating the heat equation with machine learning, solar energy systems can achieve improved efficiency, reliability, and longevity, advancing the goals of sustainable energy production.

Control of boundary-layer flows

A/Prof S. Stephen; Carslaw 525; sharon.stephen@sydney.edu.au; phone 9351-3048

This project is in the field of hydrodynamic stability of boundary-layer flows where viscous effects are important. The aim is towards understanding more fully the transition process from a laminar flow to a turbulent one. We will consider rotating flows which are relevant to the flow over a swept wing and to rotor-stator systems in a turbine engine. Experiments show that the boundary layer becomes unstable to stationary or travelling spiral vortices.

The project will investigate the effect of different surface boundary conditions on boundary-layer flows over rotating bodies. Effects such as suction, partial-slip, compliance and wall shape can be modelled. Suction, for example, is used to achieve laminar flow control on swept wings. The resulting system of governing ordinary differential equations will be solved numerically for the basic flow, determining important values such as the wall shear. The linear stability of these flows to crossflow instabilities will be investigated. These take the form of co-rotating vortices, observed in experiments, and only occur in three-dimensional boundary layers.

The flow for large Reynolds number, corresponding to large values of rotation, will be considered. In this case the boundary layer thickness will be very small so asymptotic methods of solution will be used. Different asymptotic regimes will need to be considered and solutions obtained in each region. Matching the solutions between the regimes and satisfying the boundary conditions will lead to an eigenrelation. Inviscid and viscous instability modes will be considered.

The effect of the surface boundary conditions on the disturbance wave number and wave angle will be determined. This will have applications in possible control of boundary layers as boundaries causing stabilisation of the instabilities could lead to a delay in the transition process from a laminar flow to a turbulent flow.

Geometric singular perturbation theory and its applications

Prof M. Wechselberger; Carslaw 628; martin.wechselberger@sydney.edu.au; phone 9351-3860

Projects under the supervision of Prof M. Wechselberger include research topics in the field of dynamical systems with an emphasis on the study of pattern generation of so called multiple time-scales dynamical systems.' These multi-scale systems are ubiquitous in nature and control most of our physiological rhythms. For instance, one cycle of a heartbeat consists of a long interval of quasi steady state interspersed by a very fast change of state, the beat itself. The same is true for the creation of neural action potentials. In these physiological systems, the very fast relaxation of energy leads to the notion of a relaxation oscillator and indicates physiological processes evolving on multiple timescales.

Topics could range from a theoretical study of possible multiple time-scales dynamics associated with relaxation oscillators to the analysis of a concrete physiological rhythm and algorithmic implementation of geometric singular perturbation theory.

For more information on possible topics, please have a look at, e.g., the monograph *Geometric singular perturbation theory beyond the standard form* by Prof M. Wechselberger.

Accurate numerical estimates for the Lorenz attractor

Dr C. Wormell; Carslaw 491; caroline.wormell@sydney.edu.au

The Lorenz system is a 3-dimensional ODE, well-known for its role in the discovery of chaos by Edward Lorenz in 1963. Its attractor is a butterfly-shaped fractal, bifurcated by the stable manifold of a fixed point. However, the split in the attractor caused by this stable manifold, however, makes the structure of the dynamics slightly more complicated than many other models for chaos. For this reason many important quantities associated with the Lorenz attractor's long-term behaviour (e.g. fractal dimension, Lyapunov exponents, and rate of convergence to statistical equilibrium) are not known with much precision.

The goal of this computational project is to estimate at least one of these famous quantities to floating point accuracy. To do this, we will construct a Hofbauer extension on the Lorenz system. This breaks down the dynamical structure down into a graph whose nodes are dynamical configurations ordered by increasing rarity. The long-term behaviour of the Lorenz system can then be described through eigenvalues of so-called transfer operators on this graph; we will use highly accurate FFT-based discretisations of the transfer operators to estimate these eigenvalues, and thus the interesting quantities of the Lorenz attractor.

Sources of error in computing linear response from data

Dr C. Wormell; Carslaw 491; caroline.wormell@sydney.edu.au

Chaotic dynamical systems, while deterministically unpredictable, are known to sample special probability distributions known as SRB measures over the long term. Of particular interest is how these SRB measures change when the dynamical system itself changes. The infinitesimal change in the SRB measure to an infinitestimal perturbation is known as the linear response. There have long existed formulae that notionally allow linear response to be computed from data observed from the unperturbed system, but computations using these formaule have met with only sporadic success. This is in part because the formulae appear to be highly sensitive to small errors.

This project will study one hopeful way of computing linear response formulae, via Koopman operators. In the context of both toy systems and simple models of atmospheric dynamics, we will seek to understand which kinds of error are manageable in the context of computing linear response, and which break the computations. This will either provide the basis for reliable linear response calculations in climate models and other applications, or demonstrate that it is truly impracticable.

Combining geometric and time series information in estimating dynamical resonances

Dr C. Wormell; Carslaw 491; caroline.wormell@sydney.edu.au

The long-term behaviour of many dynamical systems can be described by their Ruelle-Pollicott resonances, a set of eigenvalue-like complex numbers that describe how quickly the future state of a system decorrelates from the current state.

There are two common ways to estimate these resonances: by time series analysis (i.e. estimating a signal from dynamical system as an auto-regressive process), or by computing the eigenvalues of the operator that tries to approximate future values of simple functions of the state by other simple functions. The first method resolves resonances accurately, but is typically limited to only one or two resonances, and gives limited spatial information; the second can in theory resolve all resonances, but is often highly inaccurate. In practice, scientists often use a combination of these two approaches. This project will connect the two respective approaches mathematics: using a mixture of rigorous mathematics and computer simulations, we will explore how linking these approaches can combine their strengths, and how to most effectively construct a combined approach when estimating from finite data.

Learning with a general loss by neural networks

Prof D. X. Zhou; Carslaw 523; dingxuan.zhou@sydney.edu.au

This project aims at mathematics analysis of deep learning with a general loss function by neural networks. The objectives include analysis of the induced deep learning algorithms and numerical simulations in dealing with some practical data.

Key references:
[1] F. Cucker and D. X. Zhou, *Learning Theory: An Approximation Theory Viewpoint*, Cambridge University Press, 2007.
[2] D. X. Zhou, Deep convolutional neural networks, Wiley Encyclopedia of Electrical and Electronics Engineering, J. Webster (ed.), 2021. DOI: 10.1002/047134608X.W8424

9 Prizes and Awards

The following prizes may be awarded to Applied Mathematics Honours students of sufficient merit. Students do not need to apply for these prizes. A complete list of the prizes and scholarships offered by the School of Mathematics and Statistics can be found here.

University Medal

A University Medal is awarded at the discretion of the Faculty to the highest achieving students who, in the opinion of the Faculty, have an outstanding academic record. A student meets the minimum levels of academic performance required for the award of a University Medal if: their final honours mark SCIE4999 is equal to or greater than 90 and their WAM on entry to Honours is equal to or greater than 80. The medal is always awarded when the final honours mark SCIE4999 is 95 or higher. More than one medal may be awarded in any year.

Joye Prize in Mathematics

Awarded to the most outstanding student completing the honours program in the School of Mathematics and Statistics.

K. E. Bullen Memorial Prize

Awarded annually to the most proficient student in Applied Mathematics honours, provided that the student's work is of sufficient merit.

Barker Prize

Awarded at the honours year examination for proficiency in Pure Mathematics, Applied Mathematics or Statistics.

M. J. and M. Ashby Prize

Offered annually for the best honours project, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics, Applied Mathematics or Statistics.

Norbert Quirk Prize No IV

Awarded annually for the best project on a given mathematical subject by a student enrolled in a Fourth Year course in mathematics (Pure Mathematics, Applied Mathematics or Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics.

Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Statistics.

Chris Cannon Prize

For the best adjudged project seminar presentation of an Applied Mathematics honours student.

10 AMSI Courses

Students are welcomed to check the courses offered in January 2025 at the

- AMSI Summer School

hosted by the University of Sydney as well as the courses available via the

- Advanced Collaborative Environment (ACE).

In principle, at most one AMSI/ACE course can be taken for credit by enrolling in the unit AMSI4001. It should be noted, however, that this is only possible if very special circumstances can be demonstrated. In particular, it is not enough to show that a given AMSI/ACE course is beneficial for a student since AMSI/ACE can be completed without enrolment in AMSI4001. Furthermore, it should be stressed that enrolment in AMSI4001 can only be done in consultation with the student's supervisor and with explicit prior approvals by the Applied Mathematics honours coordinator (Prof M. Rutkowski) and the School's honours coordinator (Prof L. Paunescu).

11 Rights and Responsibilities

Applied Mathematics Honours students will have access to the following:

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail, as well as LaTeX and printing facilities for the preparation of projects.
- A photocopying account paid by the School for assembling project source material.
- After-hours access to the Carslaw building.
- A pigeon-hole in room 728.
- Participation in the School's social events.
- Class representative at School meetings.

Applied Mathematics Honours students have the following obligations:

- Regular attendance at the weekly seminars in Applied Mathematics.
- Have regular meetings with project supervisors, and meet all deadlines.
- Utilise all School resources in an ethical manner.
- Contribute towards the academic life in Applied Mathematics at the School of Mathematics and Statistics.

12 Life After Fourth Year

Postgraduate Studies

Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the School's Coordinator of Postgraduate Studies if interested in enrolling for an MPhil or PhD at the School of Mathematics and Statistics. Students who do well in Applied Mathematics Honours may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers

Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Program and Course Coordinators may also provide advice and personal references for interested students.