

Assignment 3

Your solutions should be submitted by the beginning of the lecture on

Tuesday, 4 October 2011.

Please attach a cover sheet!

Q1 Let X be a normed space over Λ and Y be a closed subspace of finite co-dimension.

Let $f: X \rightarrow \Lambda$ be a linear functional with the property that $f|_Y: Y \rightarrow \Lambda$ is continuous.

Show that f is continuous.

Q2 Let X be a normed space over Λ , and $f: X \rightarrow \Lambda$ be a non-zero linear functional on X .

Show that the following are equivalent:

- (a) $f \notin X^*$,
- (b) $f(B(X)) = \Lambda$,
- (c) $\ker f$ is dense in X .

Q3 Let K be a subset of l_p , where $1 \leq p < \infty$. Show that the following are equivalent:

- (a) K is compact;
- (b) K is closed and for each $\varepsilon > 0$, there exist $y_1, \dots, y_{n(\varepsilon)} \in K$, such that

$$K \subseteq \bigcup_{k=1}^{n(\varepsilon)} B(y_k; \varepsilon);$$

- (c) K is closed, bounded and for each $\varepsilon > 0$, there exists $m = m(\varepsilon)$, such that

$$\sum_{k=m+1}^{\infty} |x_k|^p < \varepsilon$$

for all $x = (x_k)_{k=1}^{\infty} \in K$.