Assignment 4

This assignment should be returned by the beginning of the lecture on Friday, 5 October 2007.

Q1 Define a continuous, bijective map $f\colon D^2 \to \mathbb{C}$ which has a continuous inverse. Verify that this map is not holomorphic. Is this map conformal?

Q2 If z_0 is real and U is symmetric with respect to the real axis, prove by uniqueness that the function f in the statement of the Riemann Mapping Theorem satisfies

$$f(\bar{z}) = \bar{f}(z).$$

Q3 Let

$$f(z) = \frac{z-i}{z+i}.$$

For the functions $\log f$ and $\sqrt{f}$ give an explicit definition of a single-valued branch in a suitable region using the method in the proof of Corollary 2.3. Use the equation $f(z) = e^{F(z)+\log f(z_0)}$ to deduce a known identity (and hence to verify your answer).

Q4 Show that the family of functions $f_n(z) = z^n$, where n is a positive integer, is a normal family in $D^2 = \{z \in \mathbb{C} : |z| < 1\}$, but not in any domain that contains a point in its complement. What can be said about the family (f_n')?

Q5 Let $\mathfrak{F}$ be a family of holomorphic functions which is not normal in U. Show that there is a point $z_0 \in U$ such that $\mathfrak{F}$ is not normal in any neighbourhood of z_0. *Hint:* A compactness argument.