Objectives for Math 241

1. Midterm 1

(1) Describe sets in Cartesian coordinates; find the distance between points.
(2) Find the vector between two points; perform basic vector arithmetic (addition, scalar multiplication & magnitude), algebraically or geometrically.
(3) Find the dot product of two vectors, algebraically or geometrically; use its properties.
(4) Use the formula $\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}|\,|\mathbf{w}| \cos\theta$ to compute any one term.
(5) Find vector projection and scalar projection, algebraically or geometrically.
(6) Given a point & vector, find the parametric/symmetric equation of the parallel line, or vice versa.
(7) Given a point & vector, find the equation of the normal plane, or vice versa.
(8) Find the point in a line or plane closest to a given point.
(9) Find the cross product of vectors, algebraically or geometrically; use its properties.
(10) Find the area of a parallelogram or the volume of a parallelepiped.
(11) Find the domain of a function.
(12) Match the equations, graphs and/or contour maps of functions on $\mathbb{R}^2$; sketch simple examples.
(13) Match the equations and level sets/contour maps of functions on $\mathbb{R}^3$.
(14) Recognize, define, and sketch: cones, ellipsoids, hyperboloids of one or two sheets, elliptic paraboloids, hyperbolic paraboloids, and cylinders (of any type).
(15) Define the limit of a function of two variables.
(16) Find the limit of a function of two variables or show that it doesn't exist; use polar coordinates, the squeeze theorem and/or its value along curves.
(17) Determine where a function is continuous.
(18) Find (higher) partial derivatives from an equation, graph, contour map, or table.
(19) State and apply Clairaut's theorem; verify it.
(20) Check if a function satisfies a partial differential equation.
(21) Determine if a function is differentiable at a point; compute its differential.
(22) Find the tangent plane to the graph of a function at a point.
(23) Use linear approximation to estimate the value of a function at a point.
(24) Use the chain rule to compute partial derivatives of composite functions.

Note:

- Students are also expected to have mastery of material covered in previous courses.
- Questions on exams may combine several objectives.
- This class is cumulative; exams emphasize recent objectives but many problems rely on earlier material.

2. Midterm 2

(1) Find directional derivatives of functions along unit vectors, algebraically or geometrically.
(2) Find the gradient of a function, algebraically or geometrically.
(3) Find the direction & rate of maximal increase of functions, algebraically or geometrically.
(4) Find the tangent line or plane to the level set of a function at a point.
(5) Find the critical points of a function, algebraically or geometrically.
(6) Determine if a critical point of a function on $\mathbb{R}^2$ is a local minimum, local maximum, or saddle point, algebraically (using the Second Derivative test) or geometrically.
(7) Determine if a region in $\mathbb{R}^2$ is closed and/or bounded; find its boundary.
(8) State and apply the Extreme Value Theorem.
(9) Find the maxima of a function on a region in $\mathbb{R}^n$, even if it isn't closed & bounded.
(10) State and apply the Lagrange Multipliers Theorem.
(11) Use Lagrange multipliers to find maxima of a function on a level set.
(12) Match the parametric equation of a plane or space curve with its graph; parameterize lines, circles, ellipses, helices, and graphs of functions.
(13) Find the velocity, speed, acceleration, and tangent line of a parameterized curve.
(14) Find the length of a curve.
(15) Integrate a function over a curve (or estimate it algebraically or geometrically); find total mass, center of mass, moment of inertia, and average value.
(16) Match vector fields with their plots; plot simple examples.
(17) Match vector fields with solutions of their flow.
(18) Integrate a vector field along a parameterized curve (or estimate it algebraically or geometrically); find work.
(19) Describe orientations on curves; integrate vector fields along oriented curves.
(20) State the Fundamental Theorem of Line Integrals; verify it.
(21) Use the Fundamental Theorem of Line Integrals to integrate a (locally) conservative vector field along an oriented curve.
(22) Determine if a region in $\mathbb{R}^n$ is open and/or connected; find its interior.
(23) Find the potential of a conservative vector field.
(24) Determine if curve is closed and/or simple.
(25) Determine if a region in $\mathbb{R}^2$ is simply connected.
(26) Determine if a vector field $\mathbf{F} = (P, Q)$ on a region in $\mathbb{R}^2$ is conservative; consider where it's defined, path independence, and/or $\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}$.

3. Midterm 3

(1) Integrate functions over rectangles; find volume under graphs & average values.
(2) Estimate integrals of functions from tables, equations, or graphs.
(3) State and apply Fubini's theorem.
(4) Integrate functions over regions in $\mathbb{R}^2$ using Cartesian coordinates; compute volumes.
(5) Integrate functions over a polar rectangles using polar coordinates.
(6) Integrate functions over regions in R^2 using polar coordinates.
(7) Compute total mass, center of mass, and moments of inertia for thin plates.
(8) Integrate functions over regions in $\mathbb{R}^3$ using Cartesian coordinates.
(9) Switch the order of integration and/or match integral with sketch of region.
(10) Compute volume, center of mass, and moments of inertia of solids; find averages.
(11) Convert freely between Cartesian, cylindrical, and spherical coordinates.
(12) Integrate functions over regions in $\mathbb{R}^3$ using cylindrical coordinates.
(13) Integrate functions over regions in $\mathbb{R}^3$ using spherical coordinates.
(14) Calculate the Jacobian of a transformation.
(15) Find the image of a region under a transformation *or* find a transformation taking one region to another.
(16) Integrate a function over a region with a general change of coordinates.
(17) Match a surface with its parametrization; parameterize (subsets of) planes, spheres, cylinders, surfaces of revolution, and graphs (in any orientation).
(18) Find the tangent plane to a (parameterized) surface.
(19) Integrate a function over a surface (or estimate it algebraically or geometrically); find surface area, mass, and average value.
(20) Describe & orient the curve(s) bounding a region in $\mathbb{R}^2$.
(21) State Green's Theorem; verify it.
(22) Use Green's theorem to integrate a vector field along an oriented plane curve, even if the vector field has singularities or the curve is not closed.
(23) Use Green's theorem integrate a function over a region in $\mathbb{R}^2$, e.g., find area.

4. Final exam

(1) Compute the divergence of a vector field.
(2) Compute the curl of a vector field.
(3) Determine if a vector field on $\mathbb{R}^3$ is conservative; consider curl and line integrals.
(4) Determine if a vector field on $\mathbb{R}^3$ is the curl of some other field; consider divergence.
(5) Integrate the flux of a vector field across a plane curve (or estimate it algebraically or geometrically); find fluid flow.
(6) Use Green's theorem to compute the flux of a vector field across a plane curve.
(7) Estimate the divergence of a vector field geometrically.
(8) Estimate the curl of vector field geometrically.
(9) Determine if a surface is orientable; describe the orientations.
(10) Integrate the flux of a vector field across an oriented surface (or estimate it algebraically or geometrically); find fluid flow.
(11) Describe and orient the curve(s) on the boundary of an oriented surface.
(12) State Stoke's theorem; verify it.
(13) Use Stoke's theorem to integrate a vector field along an oriented curve.
(14) Use Stoke's theorem to integrate the curl of a vector field across an oriented surface.
(15) Describe & orient the surface(s) bounding a region in $\mathbb{R}^3$.
(16) State the Divergence Theorem; verify it.
(17) Use the Divergence Theorem to find the flux of a vector field across an oriented surface, even when the surface has boundary, or the vector field has singulatiries.