Tuesday, January 15 $\ast\ast$ *A review of some important calculus topics*

1. Chain Rule:

   (a) Let $h(t) = \sin\big(\cos(\tan t)\big)$. Find the derivative with respect to t.

   (b) Let $s(x) = \sqrt[4]{x}$ where $x(t) = \ln\big(f(t)\big)$ and $f(t)$ is a differentiable function. Find $\dfrac{ds}{dt}$.

2. Parameterized curves:

   (a) Describe and sketch the curve given parametrically by

   $$\begin{cases} x = 5\sin(3t) \\ y = 3\cos(3t) \end{cases} \qquad \text{for} \quad 0 \le t < \frac{2\pi}{3}.$$

   What happens if we instead allow t to vary between 0 and 2π?

   (b) Set up, but **do not evaluate** an integral that calculates the arc length of the curve described in part (a).

   (c) Consider the equation $x^2 + y^2 = 16$. Graph the set of solutions of this equation in $\mathbb{R}^2$ and find a parameterization that traverses the curve once counterclockwise.

3. 1st and 2nd Derivative Tests:

   (a) Use the 2nd Derivative Test to classify the critical numbers of the function $f(x) = x^4 - 8x^2 + 10$.

   (b) Use the 1st Derivative Test and find the extrema of $h(s) = s^4 + 4s^3 - 1$.

   (c) Explain why the 2nd Derivative test is unable to classify all the critical numbers of $h(s) = s^4 + 4s^3 - 1$.

4. Consider the function $f(x) = x^2 e^{-x}$.

   (a) Find the best linear approximation to f at $x = 0$.

   (b) Compute the second-order Taylor polynomial at $x = 0$.

   (c) Explain how the second-order Taylor polynomial at $x = 0$ demonstrates that f must have a local minimum at $x = 0$.

5. Consider the integral $\displaystyle\int_0^{\sqrt{3\pi}} 2x\cos(x^2)\,dx$.

   (a) Sketch the area in the xy-plane that is implicitly defined by this integral.

   (b) To evaluate, you will need to perform a substitution. Choose a proper $u = f(x)$ and rewrite the integral in terms of u. Sketch the area in the uv-plane that is implicitly defined by this integral.

   (c) Evaluate the integral $\displaystyle\int_0^{\sqrt{3\pi}} 2x\cos(x^2)\,dx$.