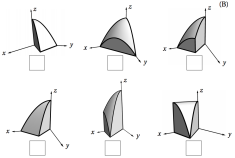
Which practice problem should we go over?

(a)

3. Label the boxes below the solid regions corresponding to the two integrals at right. **(2 points each)**

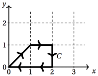
(A) $\displaystyle\int_0^1\int_y^1\int_0^{2-x^2-y^2} f(x,y,z)\,dz\,dx\,dy$

(B) $\displaystyle\int_0^1\int_0^{1-x}\int_0^{1-x^2-y^2} g(x,y,z)\,dz\,dy\,dx$



(b)

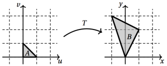
5. Let C be the oriented curve shown at right against a dashed grid of unit squares. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle y^2 + \cos x,\ x + e^y \rangle$. **(6 points)**



$\int_C \mathbf{F}\cdot d\mathbf{r} =$

(c)

8. Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking the triangle A with vertices $(0,0), (0,1), (1,0)$ to the triangle B with vertices $(0,0), (1,2), (-1,3)$. **(2 points)**



$T(u,v) = (\quad , \quad)$

Music to cheer us up: The Laughing Gnome (by David Bowie, before getting famous)

About the exam

- Consider the following integral:

$$\int_a^b \int_c^d g(x,y)\ dy\ dx.$$

  - The function $g(x,y)$ is called the *integrand*.
  - Some questions will specifically ask you to set up the integral or find the integrand. Don't waste your time evaluating the integral if you're not required to.
  - Read carefully!
- Extra office hours today 2–3:20pm.
- More office hours tomorrow 11–12:20pm.
- No lecture on Wednesday.

Remarks on integrals over curves and surfaces

- For integrating over curves, we use the unit tangent vector.
  - At a given point, there are only two choices of unit tangent vector.
  - A choice of orientation on the curve tells us which one to pick.
- But over a surface, there are infinitely many unit tangent vectors attached to any point (all in the tangent plane of that point).
  - But there aren't infinitely many choices of **normal vectors** to the tangent plane: there are only two—$\mathbf{r}_u \times \mathbf{r}_v$ and $\mathbf{r}_v \times \mathbf{r}_u = -\mathbf{r}_u \times \mathbf{r}_v$.
  - A choice of orientation on the surface tells us which one to pick at each point.
  - For surfaces, there isn't always a good choice! We'll see examples. We will only be able to integrate vector fields over surfaces with orientation.

Practice with surface orientation

Take a strip of paper, and tape the ends together (without twisting) to form a cylinder. Is it orientable?

(a) Yes.

(b) No.

(c) I don't know.

(d) I'm so excited to find out what happens when we twist the paper that I can't focus on this question.

(e) I can't answer this question, because I don't have any tape or imagination.

Practice with surface orientation

Now tape the ends of the paper together with a half-twist. This is a **Möbius** strip. Is it orientable?

(a) Yes.

(b) No.

(c) I don't know.

Practice with integrating over a surface

Let S be the graph of a function $f : D \to \mathbb{R}$, oriented upward, and let $\mathbf{F}$ be a continous vector field on S. Find a formula for $\iint_S \mathbf{F} \cdot d\mathbf{S}$ as a double integral over D.

Step 1: Parametrize S
We know what to do for graphs of functions:

$$\mathbf{r}(u, v) = \langle u, v, f(u, v)\rangle, \qquad (u, v) \in D.$$

Step 2: Find $\mathbf{r}_u \times \mathbf{r}_v$ and compare it to $\mathbf{n}$
The first part is also probably review:

$$\mathbf{r}_u = \langle 1, 0, f_x\rangle;$$
$$\mathbf{r}_v = \langle 0, 1, f_y\rangle.$$

So

$$\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & f_x \\ 0 & 1 & f_y \end{vmatrix}$$
$$= \mathbf{i}(-f_x) - \mathbf{j}(f_y) + \mathbf{k}(1)$$
$$= \langle -f_x, -f_y, 1 \rangle.$$

Compare $\mathbf{r}_u \times \mathbf{r}_v$ to $\mathbf{n}$, recalling that S is oriented upward.

(a) $\mathbf{r}_u \times \mathbf{r}_v$ is positively oriented.

(b) $\mathbf{r}_u \times \mathbf{r}_v$ is negatively oriented.

(c) I don't know.

Step 3: calculate the integral

Working with your neighbour, find a formula for the integrand $g(u, v)$ to write

$$\iint_S \mathbf{F}(x, y, z) \cdot d\mathbf{S} = \iint_D g(u, v)\ dA.$$

(a) We're working on it.
(b) We're stuck.
(c) We have two answers and we don't know which is right.
(d) We're done!