Last time: Green's theorem

Let $D \subset \mathbb{R}^2$ be such that ∂D is formed of one or more simple closed curves. Suppose $\mathbf{F} = \langle P, Q \rangle$ is a vector field such that P and Q have continuous first order partial derivatives. Then

$$\iint_D (Q_x - P_y) dA = \int_{\partial D} P dx + Q dy = \int_{\partial D} \mathbf{F} \cdot d\mathbf{r}.$$

Consider the picture on the chalkboard with paths C_1, C_2, C_3. Suppose that $Q_x = P_y$ on D and also suppose that $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 3$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1$. What is $\int_{C_3} \mathbf{F} \cdot d\mathbf{r}$?

(a) 0

(b) 2

(c) 4

(d) We don't have enough information.

Practice with curl

Find $\operatorname{curl}\langle e^x, z\cos y, \sin y\rangle$. How many of the three components are 0?

(a) 0

(b) 1

(c) 2

(d) 3

(e) I don't know.

$\operatorname{curl}\langle e^x, z\cos y, \sin y\rangle = \langle \cos y - \cos y, 0-0, 0-0\rangle = \langle 0,0,0\rangle$

Practice with curl

Find curl$\langle P(x,y), Q(x,y), 0\rangle$.

Solution

$$\begin{aligned}\text{curl}\langle P(x,y), Q(x,y), 0\rangle &= \langle \frac{\partial 0}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial 0}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\rangle \\ &= \langle 0, 0, Q_x - P_y\rangle.\end{aligned}$$

Practice with conservative vector fields and curl

Is $\mathbf{F}(x, y, z) = \langle xz, xyz, -y^2 \rangle$ conservative?
Is $\mathbf{G}(x, y, z) = \langle e^x, z \cos y, \sin y \rangle$ conservative?

(a) Yes and yes.

(b) Yes and no.

(c) No and yes.

(d) No and no.

(e) Not enough information.