# Last time: triple integrals

Let $E$ be the solid bounded by the cylinder $x^y + y^2 = 1$, the paraboloid $z = 1 - x^2 - y^2$, and the plane $z = 2$. Sketch $E$, and find a region $D$ and functions $u_1(x, y)$, $u_2(x, y)$ such that

$$E = \{(x, y, z) \mid (x, y) \in D,\ u_1(x, y) \leq z \leq u_2(x, y)\}.$$

Discuss with your neighbour(s).

(a) We're working on it.

(b) We're stuck.

(c) We have different answers and we don't know who is right.

(d) We have the same answer.

If you're done, remember that we can find the volume of $E$ by integrating the function $f = 1$ over $E$:

$$V(E) = \iiint_E dV = \iint_D \int_{u_1(x,y)}^{u_2(x,y)} dz\ dA.$$

Can you calculate $V(E)$?

# Solution

We should take

$$D = \{(x, y) \mid x^2 + y^2 \leq 1\}$$
$$u_1(x, y) = 1 - x^2 - y^2$$
$$u_2(x, y) = 2.$$

# Announcements

- Deadline for Midterm 2 regrade requests is Thursday.
- I-clicker participation is now optional (your i-clicker scores will only count towards your final grade if it helps you; otherwise they will not count).
- Quiz 4 is on Thursday.

# Practice with cylindrical coordinates

Recall

$$E = \{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1, 1 - r^2 \leq z \leq 2\}.$$

Set up the integral to find the moment of inertia about the $z$-axis of a solid with shape $E$ and constant density of $\rho_0$.

$$\begin{aligned} I_z &= \iiint_E (x^2 + y^2)\rho_0 \, dV \\ &= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^2 ((r\cos\theta)^2 + r\sin\theta)^2) r\rho_0 \, dz dr d\theta \\ &= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^2 r^3 \rho_0 \, dz dr d\theta. \end{aligned}$$

# Calculating the integral

We calculate

$$\begin{aligned}
\int_0^{2\pi}\int_0^1\int_{1-r^2}^2 r^3\rho_0\,dz\,dr\,d\theta &= \int_0^{2\pi} d\theta \int_0^1 r^3\rho_0[z]_{1-r^2}^2\,dr \\
&= 2\pi\int_0^1 \rho_0 r^3(r^2+1)\,dr \\
&= 2\pi\rho_0\int_0^1 r^5+r^3\,dr \\
&= 2\pi\rho_0\left[\frac{1}{6}r^6+\frac{1}{4}r^4\right]_0^1 \\
&= \frac{5}{6}\pi\rho_0.
\end{aligned}$$

# Practice with spherical coordinates

Consider the set

$$B = \{(\rho, \theta, \phi) \mid \rho = 1\}.$$

Can you sketch this shape? Can you write the equation in rectangular coordinates (i.e. $(x, y, z)$)? Discuss with your neighbour(s).

(a) We're working on it.

(b) We're stuck.

(c) We have different answers and we don't know who is right.

(d) We have the same answer.