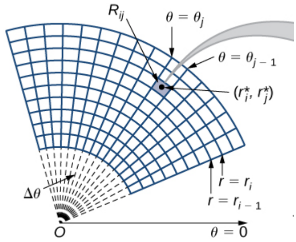
An polar rectangle R is given by the bounds $a \leq r \leq b$ and $\alpha \leq \theta \leq \beta$. Suppose we have a continuous function $f(x, y) \geq 0$ on R. We want to find the volume of the solid under the graph of f, over the region R.

We begin to approximate this volume it by dividing R into small polar rectangles R_{ij}: we divide $[a, b]$ into subintervals $[r_{i-1}, r_i]$ of length $\Delta r = \frac{b-a}{m}$, and we divide $[\alpha, \beta]$ into subintervals $[\theta_{j-1}, \theta_j]$ of length $\Delta\theta = \frac{\beta-\alpha}{n}$.

We want to know the area of each one of these rectangles. **Use the fact that a segment of a circle of radius r with central angle θ has area $\frac{1}{2}r^2\theta$ to find the area ΔA_i of R_{ij}.**



(a) There are too many words on this slide, I don't know what to do.

(b) I know what to do, but I'd still like to say I think there are too many words on this slide.

(c) $\frac{1}{2}(\Delta r)^2\Delta\theta$

(d) $\frac{1}{2}(r_i + r_{i-1})\Delta r\Delta\theta$

Integrating over polar rectangles

So to approximate the area of the solid under the graph of f over all of R:

1. We choose points (x_{ij}^*, y_{ij}^*) in each polar rectangle R_{ij}.
2. We approximate the solid by putting together all these little "polar rectangular prisms", each with base R_{ij} and height $f(x_{ij}^*, y_{ij}^*)$.
   - So each of these has volume $f(x_{ij}^*, y_{ij}^*)\Delta A_i$.
3. We say that the volume of the solid is approximately the sum of the volumes of these boxes.

Then we take the limit as the number of boxes gets large, to get better and better approximations.

Practice integrating over polar rectangles

Let $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 4 \text{ and } 0 \leq y\}$. What is

$$\iint_D ydA?$$

Since D is a polar rectangle of the form with $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi$, we have

$$\begin{aligned}
\iint_D ydA &= \int_0^\pi \int_1^2 (r \sin\theta) r dr d\theta \\
&= \int_0^\pi \int_1^2 r^2 \sin\theta dr d\theta \\
&= \int_0^\pi [\frac{1}{3} r^3 \sin\theta]_1^2 d\theta \\
&= \int_0^\pi \frac{7}{3} \sin\theta d\theta \\
&= [\frac{-7}{3} \cos\theta]_0^\pi = \frac{14}{3}.
\end{aligned}$$