MATH 595 Tuesday 13 March

Higher direct image sheaves; flat modules and flat morphisms

(1) **Chapter III, Exercises 8.1 and 8.2.**

(a) Let $f : X \to Y$ be a continuous map of topological spaces. Let $\mathscr{F}$ be a sheaf of abelian groups on X with the property that $R^i f_*(\mathscr{F}) = 0$ for all $i > 0$. Prove that for all $i \geq 0$,

$$H^i(X, \mathscr{F}) \cong H^i(Y, f_*\mathscr{F}).$$

(b) Now assume that $f : X \to Y$ is an affine morphism of schemes with X noetherian. Let $\mathscr{F}$ be any quasicoherent sheaf on X. Use the above to provide an alternate proof that

$$H^i(X, \mathscr{F}) \cong H^i(Y, f_*\mathscr{F})$$

for all $i \geq 0$. (You proved this once before using Čech cohomology, under the assumption that X and Y were both noetherian and separated.)

(2) **Chapter III, Exercise 9.4.** *The open nature of flatness*

Let X, Y be noetherian schemes, and let $f : X \to Y$ be a morphism of finite type. Let X° be the set $\{x \in X \mid f \text{ is flat at } x\}$. Prove that X° is open in X (possibly empty).