MATH 402 Worksheet 2

Friday 14 September, 2018

Exercise 1. On your homework, you proved that in Euclidean geometry, the sum of the interior angles of a triangle ("the angle sum" of the triangle) is always equal to 180°. However, in this exercise you will use a different method to show, without using the parallel postulate, that the angle sum is always $\leq 180°$. Since your proof will be independent of the parallel postulate, it will hold in non-Euclidean geometry too.

a. Suppose that you have a triangle ΔABC. Label the angle at B by β. Let M be the midpoint of $\overline{AC}$, and draw a segment $\overline{BE}$ through the points B and M so that M is also the midpoint of $\overline{BE}$. After you draw this picture, label all of the angles, using repeated labels whenever you know that two angles are congruent.

b. Prove that the angle sum of ΔBCE is equal to that of ΔABC.

c. Prove that on of the angles of ΔBCE (either the angle at B or the angle at E) is less than or equal to half of the angle β.

d. Repeating the above construction, conclude that for any natural number n you can find a triangle ΔXYZ whose angle sum is equal to that of ΔABC, but which has an angle which is less than $\frac{1}{2^n}\beta$.

e. Recall that on your homework you proved that the sum of any two interior angles of a triangle must be strictly less than 180°. Use part (d) to show that if ΔABC has angle sum strictly greater than 180°, say $180° + r$, you can construct a triangle ΔXYZ which has two angles which have angle sum larger than 180°. This contradicts the result from your homework, so you have shown that there cannot be any triangles with angle sum $> 180°$.

Exercise 2. In this exercise you will see how you can use similar triangles to prove Pythagoras' theorem. (This exercise takes place in Euclidean geometry—remember that our results like SAS and AAA similarity depended on the Parallel Postulate.)

a. Let ΔABC be a right triangle, with right angle at C, angle α at A and angle β at B. Draw a perpendicular line to $\overleftrightarrow{AB}$ through C. Label the point where this line intersects $\overleftrightarrow{AB}$ by D.

b. Label all the angles in your triangles with their measures.

c. Prove that there are two new triangles in your picture which are similar to the original triangle.

d. Look at the two triangles which both have $\overline{BC}$ as a side. Use the fact that these two triangles are similar to write down a formula for $(BC)^2$ in terms of the length of two other segments.

e. Similarly, write down a formula for $(AC)^2$.

f. Finish the proof that $(AC)^2 + (BC)^2 = (AB)^2$.

Exercise 3. Show that Playfair's postulate is equivalent to the following statement:

> **(*):** *If a line intersects but is not coincident with one of two parallel lines, then it must also intersect the other line.*

(To prove that they are equivalent, you should prove that Playfair's postulate implies **(*)**, and also that **(*)** implies Playfair's postulate.)

You do not need to hand your work in, but you are expected to complete it. If you get stuck or are unsure about your answers, come to office hours. This material is examinable and will not be covered in ordinary lecture format, so you must make sure that you understand it as it is presented here.