MATH 402 Review for September 17–24

Topics: Circle geometry (2.6, 2.7); coordinate geometry (3.1, 3.2, 3.4).
These were covered in lecture and in the Project (2.7). This material will also appear in Homework 4. Note that this summary also includes the material of Monday September 24.

1. **Recall from last week:**
   (a) Triangle similarity results: AAA, SAS, SSS. (Remember a key tool in the proof: a line meeting two sides of a triangle cuts them into proportional segments if and only if it is parallel to the third side.)
   (b) We learned about area in Euclidean geometry.

2. **Things to know about circles:**
   (a) Definitions: major and minor arc, inscribed angle, central angle, chord, diameter, semi-circle, tangent line, orthogonal circles. The power of a point with respect to a circle. The inverse of a point with respect to a circle.
   (b) Given three distinct points, there is a unique circle passing through them.
   (c) The measure of an angle inscribed in a circle is equal to half that of the corresponding central angle.
      - Use this to prove that (a) any two inscribed angles with the same arc are congruent, and (b) an angle inscribed in a semi-circle is always a right angle.

   (d) Given a point P on a circle c with centre O, the tangent line to c at P is the line through P perpendicular to the ray $\overrightarrow{OP}$.
   (e) Given a point P outside of a circle c, you can construct exactly two points on c such that the tangents to c at those points pass through P.
   (f) Given two distinct points P, Q inside a circle c, there is a unique circle c' which passes through P and Q and which is orthogonal to c.

3. **Things to know about coordinate geometry:**
   (a) Definitions: coordinates (x, y), four quadrants, measure of angles (in degrees or radians, measured counterclockwise), definition of trigonometric functions.
   (b) Know the formulas for a line, a circle, and distance between two points.
   (c) Use properties of circles to prove the Law of Sines.
   (d) A vector v can be represented by a directed segment with tail A and head B. We can always assume that $A = (0, 0)$, in which case the vector v is determined by the point $B = (x, y)$. We write $v = \vec{(x, y)}$.
   (e) Know how to add and scale vectors.

Practice Questions

1. Something to notice: if you have a formula that involves the product of the lengths of two segments, it often comes (a) from from knowing the area of something, or (b) from knowing that two fractions are equal using a similarity argument, and then multiplying through by the denominators. Look for examples of this from lecture or homework (not just in this week's material).

2. **Practice with coordinate geometry:**

   - Write down coordinates for any two points. Write down the equation of the line that passes through both of them. Compute the distance between them. Write down the equation of the circle that has one of them as its centre and passes through the other one.
   - In class I quickly sketched a proof that the midpoint of a segment AB is the point M such that $\vec{M} = \frac{1}{2}(\vec{A} + \vec{B})$. Think about how you would fill in the details of this proof.

3. **Practice with inscribed and central angles:**

   - Draw a circle in Geometry Explorer. Draw an inscribed angle (inscribed along a major arc). Draw a bunch of inscribed angles along the same arc. Check that they have the same measure. Draw the central angle (it's the same for all of them). Check that its measure is twice as big as the inscribed angles.