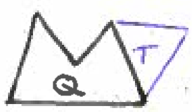
(HW 11 ex 1)

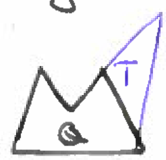
**Question:** ①

- Let $Q$ be an $n$-gon, not necessarily g. convex, and suppose you add a triangle $T$ to $Q$ by identifying a side of $T$ with a side of $Q$.

  Depending on whether the two other sides of $T$ form $180°$ angles with the sides of $Q$, one obtains an $n+1$-gon, $n$-gon or $(n-1)$-gon $P$




$n+1$




$n$

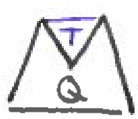

$n-1$

  Show that in each case, the defect of $P$ is

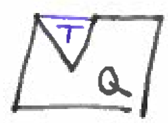
  defect $(T)$ + defect $(Q)$.

- Now suppose you add a triangle $T$ to $Q$ s.t. two of $T$'s sides are identified with sides of $Q$.

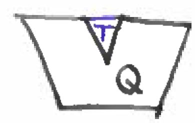
  The resulting polygon $P$ has $n-1$, $n-2$ or $n-3$ sides.




$n-1$




$n-2$



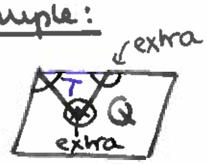

$n-3$.

  Show that in each case, the defect of $P$ is

  defect $(Q)$ + defect $(T)$.

  (you don't need to check them all! just until you feel confident)

**Example:**




$P$ has $n-2$ sides.

$$\text{Defect}(P) = 180(n-\cancel{2}4) - (\text{angle sum of } P)$$

$$= 180(n-4) - [(\text{angle sum of } Q) + (\text{angle sum of } T) - 360 - 180]$$

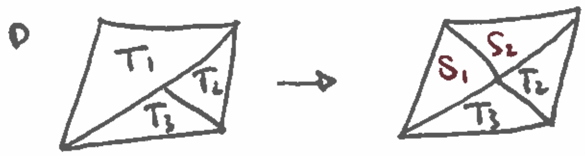
$$= 180(n-1) - [\text{angle sum of } Q + \text{angle sum } T]$$

$$= (180(n-2) - [\text{angle sum of } Q]) + (180 - [\text{angle sum of } T])$$

$$= \text{defect } Q + \text{defect } T.$$

Now given any polygon $P$ which has been divided into triangles $T_1, \dots, T_n$, we want to show $\text{defect}(P) = \sum_{i=1}^{n} \text{defect}(T_i)$

WLOG assume that all vertices in the triangulation are common vertices (if not, add more lines)



Then we'll show
$\text{defect}(T_1) = \text{defect}(S_1) + \text{defect}(S_2)$
and $\text{defect}(P) = \text{defect}(S_1) + \text{defect}(S_2) + \text{defect}(T_2) + \text{defect}(T_3)$,
which implies $\text{defect}(P) = \sum_{i=1}^{3} \text{defect}(T_i)$

A triangle in such a triangulation shares exactly 1, 2 or 3 sides with other triangles (i.e. has 0, 1, or 2 sides exposed)

Now by induction on the number of triangles in the triangulation, show that $\text{defect}(P) = \sum_{i=1}^{n} \text{defect}(T_i)$.
(use results on the previous page)