Name: SOLUTIONS

netid.:

# Math 402: Exam 1

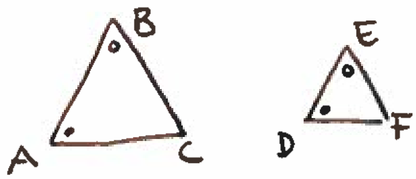
Fall semester 2018

- Do not forget to write your name and netid on top of this page.
- No notes, books, calculators, or other exam aids are allowed. You may use a ruler and colored pens or pencils if you wish.
- Turn your cell phones off and put them away. **No use of cell phones** or other communication devices during the exam is allowed.
- Write your answers **clearly and fully** on the sheets provided. If you need additional paper, raise your hand.
- **Do not tear pages** off of this exam. Doing so will be considered cheating.
- The exam consists of 6 problems and 9 pages. Check that your exam is complete.
- You have **50 minutes** to complete the exam.

**Good luck!!**

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | $\sum$ |
|---|---|---|---|---|---|---|---|
| Total possible | 5 | 10 | 35 | 15 | 20 | 15 | 100 |
| Your points | | | | | | | |

**Problem 1:** (5 Points) In class, we proved the AAA similarity theorem. Use this to prove an AA similarity theorem in Euclidean geometry.




Since angles of a triangle sum to 180°
in Euclidean geometry:

$\angle A + \angle B + \angle C = 180°$

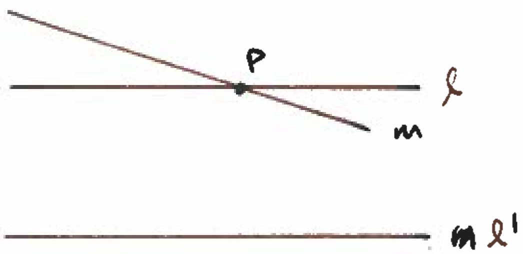
$\angle D + \angle E + \angle F = 180°$

By assumption $\angle A \cong \angle D$
$\angle B \cong \angle E$

$\therefore \angle C \cong \angle F$

By AAA similarity, $\triangle ABC \sim \triangle DEF$.

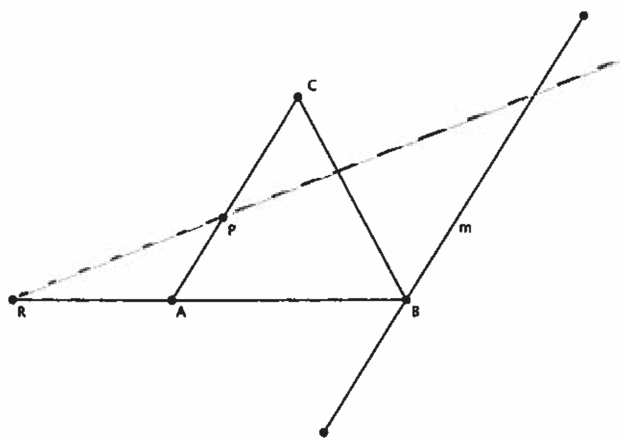
**Problem 2:** (10 Points) Suppose that $\ell$ and $\ell'$ are parallel lines. Suppose that $m$ is a line which intersects $\ell$ but is not equal to $\ell$. Use Playfair's postulate to show that $m$ intersects $\ell'$.




let P be the intersection point.
By Playfair's Postulate ~~m~~ $\ell$ is the only line passing through P parallel to $\ell'$.

So m cannot be parallel to $\ell'$.

**Problem 3:** (3 + 3 + 3 + 6 + 5 + 6 + 5 + 4 = 35 Points) Let $\Delta ABC$ be a triangle in Euclidean geometry. Extend side $\overline{AB}$ to a point $R$, and choose a point $P$ on the side $\overline{AC}$ not equal to $A$ or $C$. Let $m$ be the line through $B$ parallel to the side $\overline{AC}$, and consider the ray $\overrightarrow{RP}$.




(a) Explain why the ray $\overrightarrow{RP}$ must intersect either side $\overline{BC}$ or side $\overline{AB}$.

- Pasch's axiom.

3 (clearly $\overrightarrow{RP}$ doesn't contain A, B, or C)

(b) Now prove that the ray $\overrightarrow{RP}$ must intersect $\overline{BC}$ by showing that it cannot intersect $\overline{AB}$. Label the intersection point $Q$.

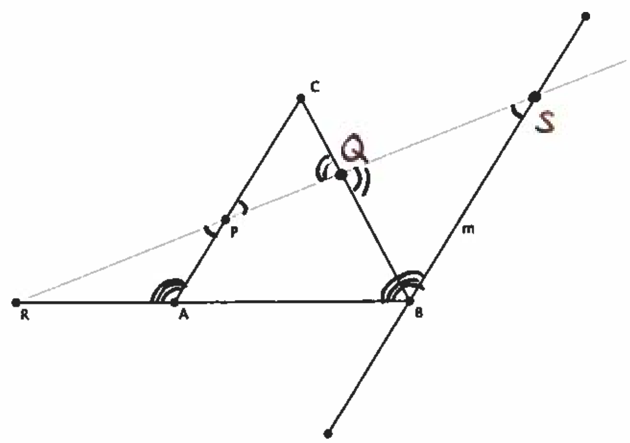
if $X \in \overline{AB} \cap \overrightarrow{RP}$, $\overleftrightarrow{RP} = \overleftrightarrow{RX} = \overleftrightarrow{AB}$ #.

3 $R \notin \overleftrightarrow{AB}$.

(c) Explain why the ray $\overrightarrow{RP}$ also intersects the line $m$. Label the intersection point $S$.

$m \parallel \overleftrightarrow{AC}$; $\overrightarrow{RP}$ intersects $\overline{AC}$.

3




(d) Prove that the triangles $\Delta RAP$ and $\Delta RBS$ are similar.

- $\angle ARP = \angle BRS$
- $\angle RAP \cong \angle RBS$ since $m \parallel \overleftrightarrow{AC}$

$\therefore$ by AA similarity, $\Delta RAP \sim \Delta RBP$

6

(e) Use the previous question to write down a formula that shows the relationship of lengths $SB$ and $RB$ to the corresponding sides of $\Delta RAP$.

5

$$\frac{SB}{PA} = \frac{RB}{RA}.$$

(f) Prove that the triangles $\Delta CPQ$ and $\Delta BSQ$ are similar.

$\angle PQC \cong \angle SQB$ by Vertical Angle Theorem.

$\angle CPQ \cong \angle BSQ$ since $m \parallel \overleftrightarrow{AC}$

6

$\therefore$ by AA similarity, $\Delta CPQ \sim \Delta BSQ$.

(g) Use the previous question to write down a formula for the length $CP$ in terms of $CQ$ and the corresponding sides of $\Delta BSQ$.

5

$$\frac{CP}{BS} = \frac{CQ}{BQ} \implies CP = \frac{(CQ)(BS)}{(BQ)}.$$

(h) Use parts (d) and (f) to prove that

$$\frac{(CP)}{(PA)}\frac{(AR)}{(RB)}\frac{(BQ)}{(QC)} = 1.$$

4

$$\frac{(CP)}{(PA)} \cdot \frac{(AR)}{(RB)} \cdot \frac{(BQ)}{(QC)} = \frac{(\cancel{CQ})(BS)}{(\cancel{BQ})} \cdot \frac{1}{(PA)} \cdot \frac{(AR)}{(RB)} \cdot \frac{(\cancel{BQ})}{(\cancel{QC})} \quad \text{(substitute from(g))}$$

$$= \frac{(BS)}{(PA)} \cdot \frac{(AR)}{(RB)}$$

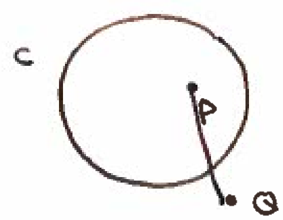
$$= 1 \quad \text{by (e)}.$$

**Problem 4:** (5 + 10 = 15 Points)

(a) Give a careful definition of (the boundary of) the circle with centre $O$ and passing through the point $P$. Recall that in Hilbert's system *distance* is not an undefined term, but *congruence* for line segments is.

The boundary of the circle consists of all points Q with $\overline{OQ} \cong \overline{OP}$.

(b) Let $c$ be a circle, and let $P$ and $Q$ be points such that the power of $P$ with respect to $c$ is $-\frac{1}{2}$ and the power of $Q$ with respect to $c$ is $\frac{1}{2}$. Does the line segment $\overline{PQ}$ intersect the boundary of the circle $c$? Draw a picture and justify your answer.




power $P < 0$ $\Rightarrow$ P is inside

power $Q > 0$ $\Rightarrow$ Q is outside

$\Rightarrow$ by Principle of circle continuity, $\overline{PQ}$ intersect the boundary of the circle.

**Problem 5:** (20 Points) In four sentences or fewer, describe the development of non-Euclidean (specifically, hyperbolic) geometry. You do not need to know dates or names of mathematicians. You should use words from the following list: *axiomatic system, postulate/axiom, independent, consistent, contradiction, model.*

Euclid established an axiomatic system for doing geometry. It had five axioms, the fifth of which seemed less obvious than the first four.

People tried to prove that this last axiom (the Parallel Postulate) followed from the first four axioms by replacing it with statements that contradict it and trying to prove theorems that contradict the first four axioms, but they did not succeed in finding contradictions.

Eventually people found models for this new axiomatic system, hyperbolic geometry, thus proving that the new axiomatic system is consistent, and that the Parallel Postulate is independent of the first four axioms.

**Problem 6:** ($3 \times 5 = 15$ Points) Are the following statements true or false? Circle the correct answer.

*No partial credit will be given.*

| | | | |
|---|---|---|---|
| (a) | Every Euclidean triangle is inscribed in a circle. | True | False |
| (b) | In neutral geometry, a line which is perpendicular to one of two parallel lines is also perpendicular to the other. | True | False |
| (c) | SAS congruence is an axiom in Hilbert's axiomatic system. | True | False |
| (d) | $x^2 + 2y^2 = 4$ is the equation of a (Euclidean) circle. | True | False |
| (e) | The measure of an inscribed angle is twice that of its corresponding central angle. | True | False |