MATH 402 Homework 6

Due Friday October 20, 2017

Exercise 1. Let $G = T_{AB} \circ r_\ell$ be a glide reflection, where the displacement vector $\vec{AB}$ is non-zero. Show that

a. the only invariant line under G is ℓ.

b. G has no fixed points.

Exercise 2.

a. Let k, ℓ, m, n be any four lines (possibly not distinct). Let f be the composition $f = r_{\ell_4} \circ r_{\ell_3} \circ r_{\ell_2} \circ r_{\ell_1}$. Prove that we can find two lines ℓ_1' and ℓ_2' such that $f = r_{\ell_1'} \circ r_{\ell_2'}$.

Hint: write $f = g \circ h$ where g and h are either translations or rotations. If one of them is a rotation with centre O, show that there is some line m through O, and some other lines n and n' such that $g = r_{n'} \circ r_m$ and $h = r_m \circ r_n$. For the case that both g and h are translations, you will need a different argument.

b. Use this to prove that an isometry is *even* if and only if it is orientation-preserving, and it is *odd* if and only if it is orientation-reversing.

Exercise 3. Prove that the composition of two glide reflections is a translation if and only if the reflection lines for the two glide reflections are parallel. What kind of transformation do you get if the reflection lines are not parallel?

Exercise 4. We have already seen that sometimes it is useful to take an operation f which we understand (such as rotation about $(0, 0)$ or reflection across the x-axis) and use it to express more difficult operations, by performing some isometry g, applying the operation f, and then undoing the isometry g: that is, we look at the composition $f^{-1} \circ g \circ f$. This is related to an important phenomenon called *conjugation*: the *conjugate* of g by f is given by $f \circ g \circ f^{-1}$. Often the conjugate has some of the same kinds of properties as the original function g.

For example, you proved on Worksheet 5 that if you conjugate a reflection by another reflection, the result is again a reflection:

$$r_\ell \circ r_m \circ r_\ell = r_{r_\ell(m)}.$$

- Generalize this to show that if f is any isometry, $f \circ r_m \circ f^{-1} = r_{f(m)}$.

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.