Algebra and Topology Special Session

Australian Mathematical Society Annual Meeting
University of Sydney, September 1998.

Titles and abstracts of the talks

Non-commutative geometry and quantum theory

Alan Carey (University of Adelaide)

Connes' non-commutative geometry appears to be the natural language for capturing invariants of non-commutative algebras (these arising for example in quantum theory). The computation of these invariants in particular examples such as the quantum Hall effect uses tools from topology. In this talk, which is a companion to the talk of Mathai Varghese, I will describe briefly the invariants associated to algebras constructed from Schrodinger operators on hyperbolic space and their computation via methods used in approaches to the Novikov conjecture.

Specht modules for Ariki-Koike algebras

Jie Du (University of New South Wales)

Specht modules for an Ariki-Koike algebra have been investigated recently in the context of cellular algebras. Thus these modules are defined as quotient modules of certain "permutation" modules, that is, defined as "cell modules" via cellular basis. So, cellular bases plays a decisive role in these work. However, the classical theory or the work for type A Hecke algebras suggest that a construction as *submodules* without using cellular bases should exist. Following our previous work, we shall introduce here Specht modules for an Ariki-Koike algebra as submodules of those "permutation" modules, generalizing some construction given by Dipper-James for type A Hecke algebras.

Local systems, logarithmic connections, and coverings of Riemann surfaces

Mark Kisin (University of Sydney)

If X is a compact Riemann surface, S a finite set of points in X, and $U = X - S$, then there is a canonical way to associate to each representation L of the fundamental group of U, a vector bundle V on X, equipped with a connection with logarithmic poles in S. We will discuss the problem of computing V, given L.

This problem has applications to the question of computing the vector bundle $f_* O_Y$ for a map $f\colon Y \to X$ of compact Riemann surfaces. We present results, obtained with E. Carberry, on how to compute $f_* O_Y$ in certain situations.

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- Kisin
- Kovács
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- G. Martin
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- Wysocki

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Lie powers of the natural module for finite general linear groups

L. G. Kovács (Australian National University)

Let L be a free Lie algebra of finite rank r over a field F. For each positive integer n, denote the degree n homogeneous component of L by L^n. The group of graded algebra automorphisms of L may be identified with $GL(r, F)$ in such a way that L^1 becomes the natural module, and then the L^n are referred to as the Lie powers of this module. When F is of characteristic zero, the $GL(r, F)$-module structure of the L^n was understood already fifty years ago. This talk will report current work on the case when F is a finite prime field.

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Counting Conjugacy Classes of Representations of a Homology 3-Sphere Group

Ben Martin (Australian National University)

Let Γ be the fundamental group of a homology 3-sphere M. Roughly speaking, Casson's invariant $\lambda(M)$ counts the number of conjugacy classes of representations of Γ into SU(2). I describe a similar procedure for counting conjugacy classes of representations of Γ into a reductive algebraic group G, using maps between representation spaces induced by restriction of representations to subgroups. For any Heegaard splitting of M, we obtain a nonnegative integer n, but it is not clear whether this defines a homology 3-sphere invariant (n may depend on the choice of splitting).

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UQR Mappings, Siegel's Theorem and the Hilbert-Smith Conjecture

Gaven Martin (University of Auckland)

We explore a fascinating circle of ideas linking the classification of local dynamics, analytic continuation properties of solutions of Beltrami systems and the Hilbert-Smith conjecture. The links are thru the study of mappings which are conformal with respect to some bounded measurable Riemannian structure. We relate various analytic properties of these maps to the solution of a central and long standing problem in Lie Theory/Topology, namely the Hilbert-Smith conjecture. We present a solution of this conjecture for quasiconformal actions.

Simple modules of Ariki-Koike algebras and affine Hecke algebras

Andrew Mathas (University of Sydney)

The *Ariki-Koike algebra* is a deformation of the group algebra of the wreath product of a cyclic group and a symmetric group. These algebras are in some sense the most important examples of the *cyclotomic Hecke algebras* introduced by Broué and Malle. The cyclotomic Hecke algebras are attached to *complex reflection groups* (in much the same way as the Iwahori–Hecke algebras are associated with Coxeter groups) and, conjecturally, these algebras play an important role in the representation theory of groups of Lie type.

In this talk we describe the classification of the irreducible modules of the Ariki-Koike algebras. It turns out that this problem is intimately connected with the the representation theory of the affine Hecke algebras of type **A** and with the representation theory of affine quantum groups. The intermarriage of these ideas leads not only to the classification of the simple modules of the Ariki-Koike algebras but also to the classification of the simple modules of the affine Hecke algebra of type **A** and to a description of the integrable highest weight modules of affine quantum groups.

This is joint work with Susumu Ariki.

Transvector algebras and elementary representations of Yangians

Alexander Molev (University of Sydney)

Let V be a finite-dimensional irreducible representation of a simple complex Lie algebra of type $X_n = A_n, B_n, C_n, D_n$. For $m \leq n$ consider the subspace $W \subset V$ of the singular vectors with respect to the subalgebra of type X_{n-m}. The corresponding transvector algebra $\mathbb{Z}(X_n, X_{n-m})$ is generated by the operators in V that preserve the subspace W. We construct an algebra homomorphism from the (twisted) Yangian $\mathbb{Y}_m$ of type X_m to the algebra $\mathbb{Z}(X_n, X_{n-m})$. This provides each weight subspace W_μ with a structure of an irreducible $\mathbb{Y}_m$-module (called elementary module). In the case of the A series we give a branching rule for the restriction of the elementary modules and their tensor products to the subalgebra $\mathbb{Y}_{m-1}$ and construct the corresponding Gelfand–Tsetlin-type bases.

Current status of the Jacobian Conjecture

Walter D. Neumann (University of Melbourne)

The n-dimensional Jacobian Conjecture posits that a polynomial map $C^n \to C^n$ is invertible if (and only if) it has constant non-zero Jacobian. Although it has had several "proofs" in the literature, it is still open, even for $n = 2$. This lecture will give a fast survey of progress on this conjecture, including some recent results for $n = 2$.

Factorisation in fractional powers

Alf van der Poorten
(Centre for Number Theory Research, Macquarie University, Sydney)

Consider an irreducible polynomial $f(x_1, x_2, \ldots, x_n)$ in several variables over a field F. Factorisation of such a polynomial in fractional powers of its variables is the same thing as factorisation in the polynomial ring $F(x_1, x_2, \ldots, x_n)$ of the polynomials $f(x_1^{q_1}, x_2^{q_2}, \ldots, x_n^{q_n})$ for arbitrary n-tuples $(q_1, q_2, \ldots, q_n)$ of positive integers. Excluding some trivial exceptions, it is a remarkable fact that there are nonetheless absolute irreducibles and that every factorisation of f in fractional powers is into a number of these factors absolutely bounded in terms of the multi-degree of f. I will report an ingenious argument producing a sharp bound on the number of factors, improving results originally due to Ritt and Gourin.

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Triangulations, hyperbolic geometry and the homeomorphism problem

Hyam Rubinstein (University of Melbourne)

This is joint work with W. Jaco. In 1995 at Montreal, Casson outlined an approach to Thurston's geometrisation conjecture for 3-manifolds. The idea was to directly put hyperbolic metrics on triangulations, using a suitable potential function which had to be minimised. Casson found a fascinating connnection with normal surface theory - an obstruction to finding such a minimum was the existence of an embedded normal 2-sphere or non peripheral torus. (He was considering first the case of ideal triangulations where there is a non empty boundary at the ideal vertices). He also observed that the existence of a (straight) hyperbolic ideal triangulation implies no such normal surfaces occur.

We have developed a theory of efficient triangulations which have such good properties, in both the ideal and closed cases. Recently we have shown that *all ideal triangulations are efficient if one is.* Moreover ideal triangulations have a better property, which we call super efficiency, namely there are no immersed normal spheres or non peripheral tori. If a compact 3-manifold with non empty boundary is irreducible and atoroidal, then it has a super efficient ideal triangulation, combining Casson's argument with Thurston's uniformisation theorem and results of Epstein and Penner. Hence *all* ideal triangulations are super efficient in this key case.

Finally we have shown that in the closed case, any two efficient triangulations can be connected by a path of triangulations where each intermediate triangulation has at most 7 vertices. If a bound could be found on the number of edges in such triangulations, in terms of the initial two efficient ones, then this would give a solution of the homeomorphism problem for 3-manifolds.

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Branches of higher dimensional algebra

Ross Street (Macquarie University)

The talk will survey recent advances in the study of higher dimensional categorical structures involving the higher operads of Michael Batanin defined in terms of plane trees.

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Dynamics and holomorphic curves in dimension three

Krzysztof Wysocki (University of Melbourne)

Let X be a nowhere vanishing vector field on a closed three-dimensional manifold M. We would like to understand the dynamics of X. One idea, going back to Poincare and Birkhoff, is to look for a global surface of section of X. By a global surface of section we mean a compact surface Σ in M whose boundary consists of periodic orbits of X , the interior of Σ is transversal to X and every orbit, other than those in $\partial\Sigma$, hits the interior of Σ in forward and backward time. Finding such a global surface of sections reduces the understanding of the dynamics of X to the problem of understanding the iterates of a self-map of a global section.

The aim of the talk is to describe how one can use the theory of pseudo-holomorphic curves in order to construct global surfaces of section for Reeb vector fields on the three-sphere.

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