Attachment to Examination Paper (Semester 1, 2000)
MATH2902 LINEAR ALGEBRA (Advanced)

Name:
Student Number:

Section A

Write your answers to this section in the places indicated.
Place the completed sheet inside the answer booklet for Section B.

A1. Consider the following functions:

$$T_1 : \mathbb{R}^2 \to \mathbb{R}^2 : (x, y) \mapsto (x, x + y)$$
$$T_2 : \mathbb{R}^2 \to \mathbb{R}^2 : (x, y) \mapsto (x + 1, y)$$
$$T_3 : \mathbb{R}^3 \to \mathbb{R}^3 : (x, y, z) \mapsto (x + z, 0, y)$$
$$T_4 : \mathbb{R} \to \mathbb{R} : x \mapsto x^3 - 1$$
$$T_5 : \mathbb{R}^2 \to \mathbb{C} : (x, y) \mapsto 2x + 3iy$$
$$T_6 : \mathrm{Mat}_{2,2}(\mathbb{R}) \to \mathbb{R} : A \mapsto \det A$$
$$T_7 : \mathrm{Mat}_{2,2}(\mathbb{R}) \to \mathrm{Mat}_{2,2}(\mathbb{R}) : A \mapsto A^2$$
$$T_8 : V \to V : p(x) \mapsto p'(x)$$

where V is the space of all polynomials over $\mathbb{R}$ and $p'(x)$ is the derivative of $p(x)$. List all of these which are:

(i) injective **Answer(s):**

(ii) surjective **Answer(s):**

(iii) bijective **Answer(s):**

(iv) linear operators **Answer(s):**

(v) nilpotent operators **Answer(s):**

(vi) linear transformations of rank 1 **Answer(s):**

(vii) linear transformations of rank 2 **Answer(s):**

(viii) linear transformations of nullity 1 **Answer(s):**

(ix) linear transformations of nullity 0 **Answer(s):**

(x) isomorphisms **Answer(s):**

A2. Consider each of the following statements. Circle **T** if you believe the statement to be true, and circle **F** if you believe it to be false. Marks will be deducted for incorrect answers.

Some questions refer to the following matrices:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad F = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(i) A is invertible as a matrix over $\mathbb{R}$ **T F**

(ii) B is invertible as a matrix over $\mathbb{Z}_{103}$ **T F**

(iii) there exists a matrix over the field $\mathbb{Z}_7$ which is invertible and nilpotent **T F**

(iv) C is diagonalizable as a matrix over $\mathbb{R}$ **T F**

(v) D is nilpotent as a matrix over $\mathbb{R}$ **T F**

(vi) there exists an invertible matrix M such that $ME = F$ **T F**

(vii) the vectors $(1,3,4,5)$, $(2,2,0,1)$, $(0,0,1,0)$ span $\mathbb{R}^4$ **T F**

(viii) the vectors $(1,3,4,5)$, $(2,2,0,1)$, $(0,0,1,0)$ in $\mathbb{R}^4$ are linearly independent **T F**

(ix) the dimension of $\langle(1,2,0),(3,2,0),(4,0,1),(5,1,0)\rangle$ is less than 3 **T F**

(x) if $p(x)$ is a non-zero polynomial over $\mathbb{R}$ and $p'(x)$ is its derivative then $p(x)$ and $p'(x)$ are linearly independent **T F**

(xi) the functions $x+1, x, x^2$ are linearly dependent in $\mathbb{R}^{\mathbb{R}}$ **T F**

(xii) there exists a basis for $\mathbb{R}^{\mathbb{R}}$ containing the functions $\cos x$ and $\cos 2x$ **T F**

(xiii) $\{f : \mathbb{R} \to \mathbb{R} | f(1) \leq f(2)\}$ is a subspace of $\mathbb{R}^{\mathbb{R}}$ **T F**

(xiv) $\{f : \mathbb{R} \to \mathbb{R} | f(1) = f(2)\}$ is a subspace of $\mathbb{R}^{\mathbb{R}}$ **T F**

(xv) $\mathbb{R}^2$ can be written as a union of one-dimensional subspaces U_1 and U_2 **T F**

(xvi) $(1,1,0)$ and $(1,0,1)$ span $\mathbb{Z}_2^3$ **T F**

(xvii) $\mathbb{R}$ is algebraically closed **T F**

(xviii) $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(0, 1/\sqrt{2}, -1/\sqrt{2})$, $(2/\sqrt{6}, -1/\sqrt{6}, -1/\sqrt{6})$ is an orthonormal basis for $\mathbb{R}^3$ **T F**

(xix) $\|u+v\| \leq \|u\| + \|v\|$ for all u, v in an inner product space **T F**

(xx) $\|u-v\| \geq \|u\| - \|v\|$ for all u, v in an inner product space **T F**

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