

# Analysis and Partial Differential Equation

University of New South Wales

19 July 2016

All talks are in The Red Centre, East Wing, Room M032

## Programme

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|-------------|--|
| 10:05–14:35 | <b>Welcome</b>   |
| 10:05–10:45 | <b>Quoc Thong Le Gia</b> (University of New South Wales)<br><i>Bayesian estimations in PDEs with random coefficients</i>         |
| 10:50–11:30 | <b>Xuan Thinh Duong</b> (Macquarie University)<br><i>A Moser type inequality for Bessel Laplace equations and applications</i>   |
| 11:35–13:10 | <b>Lunch</b>   |
| 13:10–13:50 | <b>Ben Andrews</b> (Australian National University)<br><i>Sharp gradient estimates for elliptic equations on manifolds</i>       |
| 13:55–14:35 | <b>Jiakun Liu</b> (University of Wollongong)<br><i>Dirichlet problem for Monge-Ampère type equations</i>                         |
| 14:35–15:05 | Afternoon Tea  |
| 15:05–15:45 | <b>Yong Wei</b> (Australian National University)<br><i>IMCF and Geometric Inequality on Kottler Space</i>                        |
| 15:50–16:30 | <b>Joshua Ching</b> (University of Sydney)<br><i>Liouville-type theorems for coercive elliptic equations with gradient terms</i> |

For detail see <http://www.maths.usyd.edu.au/u/PDESeminar/analysis-and-pde/2016/07/>

## Abstracts of Talks

### Sharp gradient estimates for elliptic equations on manifolds

Ben Andrews (Australian National University)

I will discuss recent work with Changwei Xiong in which we derive sharp gradient estimates for solutions of elliptic equations on manifolds, given a diameter bound and a lower bound on the Ricci curvature. We prove this using maximum principles applied to suitable two-point functions, and capture as special cases earlier results of Modica, Caffarelli-Garofalo-Segala, and Farina-Valdinoci.

### Liouville-type theorems for coercive elliptic equations with gradient terms

Joshua Ching (University of Sydney)

In this talk, we start with a brief survey of Liouville-type theorems for coercive elliptic equations with gradient terms in  $\mathbb{R}^N$ . For the model problem  $\Delta u = u^q |\nabla u|^m$ , we provide sharp conditions on  $m, q \geq 0$  so that any  $C^1(\mathbb{R}^N \setminus \{0\})$ -solution bounded near 0 is identically constant. Our technique, which improves a theorem of Farina and Serrin (2011) is different from the existing literature and relies on a local estimate and the comparison principle. We conclude with further Liouville results for a larger class of elliptic equations from my PhD thesis.

This talk also present results from Ching and Cirstea (2015, Analysis & PDE).

### A Moser type inequality for Bessel Laplace equations and applications

Xuan Thinh Duong (Macquarie University)

We study Bessel operators and Bessel Laplace equations studied by Weinstein, Huber, and related the harmonic function theory introduced by Muckenhoupt–Stein. We establish a new Moser type inequality for these harmonic functions and use it to give a direct proof for the equivalence of characterizations of the Hardy spaces associated to Bessel operator via non-tangential maximal function and radial maximal function using Poisson semigroup.

This is joint work with Ji Li (Macquarie University), Zihua Guo (Monash University) and Dongyong Yang (Xiamen University).

### Bayesian estimations in PDEs with random coefficients

Quoc Thong Le Gia (University of New South Wales)

We analyze Quasi-Monte Carlo numerical integration methods in Bayesian estimation of solutions to parametric operator equations with holomorphic dependence on the parameters. Such problems arise in numerical uncertainty quantification and in Bayesian inversion of operator equations with distributed uncertain inputs, such as uncertain coefficients, uncertain domains or uncertain source terms and boundary data.

We establish error bounds for higher order, Quasi-Monte Carlo quadrature for the Bayesian estimation. It implies, in particular, regularity of the parametric solution and of the parametric Bayesian posterior density in SPoD weighted spaces. This, in turn, implies that the Quasi-Monte Carlo quadrature methods are applicable to these problem classes, with dimension-independent convergence rates  $O(N^{-1/p})$  of  $N$ -point HoQMC approximated Bayesian estimates where  $0 < p < 1$  depends only on the sparsity class of the uncertain input in the Bayesian estimation.

This is joint work with Josef Dick (UNSW) and Robert Gantner and Christoph Schwab (ETH Zürich)

### Dirichlet problem for Monge-Ampère type equations

Jiakun Liu (University of Wollongong)

In this talk, we first recall some classical results of Dirichlet problem for standard Monge-Ampère equations. Then we present a recent result on the global  $C^{2,\alpha}$  estimates for solutions of the Dirichlet problem of Monge-Ampère equations arising in optimal transportation, where the cost function satisfies a strong Ma-Trudinger-Wang condition and the inhomogeneous term is  $C^\alpha$  and bounded away from zero and infinity. This is a joint work with Yong Huang and Feida Jiang.

### IMCF and Geometric Inequality on Kottler Space

Yong Wei (Australian National University)

The long time existence and convergence of the inverse mean curvature flow, and Minkowski-type inequality have already been established by many authors for star-shaped and strictly mean convex hypersurface in Kottler-Schwarzschild space. In this talk, we will show that the same conclusion also holds if we replace the condition “strictly mean convex” by “weakly mean convex”.