Analysis and Partial Differential Equation

Australian National University

9/10 February 2016

All talks are in the John Dedman Building in Room G35

Programme

	Tuesday 9 February
14:30–14:35	**Welcome**
14:35–15:20	**Mark Veraar** (TU Delft, visiting ANU) *A new approach to maximal regularity for parabolic PDEs*
15:25–16:00	**Afternoon Tea**
16:00–16:45	**Anna Tomskova** (University of New South Wales) *Fréchet differentiability of the norm of non-commutative L_p-spaces*
16:50–17:25	**Piotr Rybka** (University of Warsaw, Poland, visiting UOW) *Special cases of the planar least gradient problem*
19:00–21:00	**Dinner** at the pizzeria Debacle
	Wednesday 10 February
09:00–09:45	**Lixin Yan** (Sun Yat Sen University, China, visiting Macquarie) *Multicommutators and Multiplier Theorems*
09:50–10:20	**Morning Tea**
10:20–11:05	**Yanqin Fang** (Wollongong) *Liouville theorems involving the fractional Laplacian*
11:10–11:55	**Andrew Hassell** (Australian National University) *Upper and lower bounds on boundary values of Neumann eigenfunctions, and applications*

For detail see `http://www.maths.usyd.edu.au/u/PDESeminar/analysis-and-pde/2016/02/`

Abstracts of Talks

Liouville theorems involving the fractional Laplacian

Yanqin Fang (University of Wollongong)

The fractional Laplacian in Euclidean space is a non-local operator. We establish Liouville type theorems, non-existence of positive solutions of Dirichlet problem involving the fractional Laplacian. We obtain the equivalence between a partial differential equation and an integral equation from the uniqueness of harmonic functions on a half space. Applying the method of moving planes in integral forms, we prove the non-existence of positive solutions of an integral equation.

Upper and lower bounds on boundary values of Neumann eigenfunctions, and applications

Andrew Hassell (Australian National University)

For smooth bounded Euclidean domains, we prove upper and lower L^2 bounds on the boundary data of Neumann eigenfunctions, and prove quasi-orthogonality of this boundary data in a spectral window. The bounds are tight in the sense that both are independent of eigenvalue; this is achieved by working with an appropriate norm for boundary functions, which includes a 'spectral weight', that is, a function of the boundary Laplacian. This spectral weight is chosen to cancel concentration at the boundary that can happen for 'whispering gallery' type eigenfunctions.

Special cases of the planar least gradient problem

Piotr Rybka (University of Warsaw, Poland)

We study the least gradient problem in two special cases: 1. the natural boundary conditions are imposed on a part of the stricly convex domain; 2. the Dirichlet type data are imposed on a boundary of a rectangle. We show the existence of solutions and study properties of solutions for special cases of the data.

Fréchet differentiability of the norm of non-commutative L_p-spaces

Anna Tomskova (University of New South Wales)

Let M be a von Neumann algebra and let $(L_p(M), \|.\|_p)$, $1 \le p < \infty$ be Haagerup's L_p-space on M. The main ideas of the proof that the differentiability properties of $\|.\|_p$ are precisely the same as those of classical (commutative) L_p-spaces are presented. Our main instruments are the theories of multiple operator integrals and singular traces. This is joint work with Fedor Sukochev, Denis Potapov and Dimitry Zanin.

A new approach to maximal regularity for parabolic PDEs

Mark Veraar (TU Delft, Netherlands)

Maximal regularity can often be used to obtain a priori estimates which give global existence results. In this talk I will explain a new approach to maximal L^p-regularity for parabolic PDEs with time dependent generator $A(t)$. Here we do not assume any continuity properties of $A(t)$ as a function of time. We show that there is an abstract operator theoretic condition on A(t) which is sufficient to obtain maximal L^p-regularity. As an application I will obtain an optimal $L^p(L^q)$ regularity result in the case each $A(t)$ is a system of $2m$-th order elliptic differential operator on $\mathbb{R}^d$ in non-divergence form. The main novelty is that the coefficients are merely measurable in time and we allow the full range $1 < p, q < \infty$. This talk is based on joint work with Chiara Gallarati.

Multicommutators and Multiplier Theorems

Lixin Yan (Sun Yat-Sen University, China)

We obtain the boundedness of the n-th dimensional Calderón-Coifman-Journé multicommutator from $L^{p_0}(\mathbb{R}^n) \times L^{p_1}(\mathbb{R}^n) \times \cdots \times L^{p_k}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$ in the largest possible open set of indices $(\frac{1}{p_0}, \frac{1}{p_1}, \dots, \frac{1}{p_k})$ with $\frac{1}{p_0} + \frac{1}{p_1} + \cdots + \frac{1}{p_k} = \frac{1}{p}$, which is the range $\frac{1}{k+1} < p < \infty$. The proof exploits the limited smoothness of the symbol of the multicommutator via a new multilinear multiplier theorem for symbols of restricted smoothness which lie locally in certain Sobolev spaces. Our multiplier approach to this problem is a new contribution in the understanding of Calderón's commutator program. This is joint work with Loukas Grafakos, Danqing He and Hanh Van Nguyen.